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Central Bank Independence, Government Debt and the Re-Normalization of Interest Rates*

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Abstract

We develop a New Keynesian model augmented with a rich description of fiscal policy, including debt maturity structure, where two policymakers - an independent inflation-averse central bank and a (potentially) populist fiscal authority - interact strategically. Central bank independence initially improves inflation outcomes, but this results in reduced fiscal discipline and increased debt. Eventually this leads to inflation lying above pre-independence levels. Introducing a 'flight-to-safety' regime, which suppresses the interest rates households require to hold government debt, and a conventional regime, where their time preferences return to normal, allows us to explore how changes in the natural rate can dramatically affect debt dynamics and inflation outcomes. The model offers an explanation of the build-up of government debt since the financial crisis and the subsequent emergence of significant inflation.

Keywords: New Keynesian Model; Central Bank Independence; Government Debt; Monetary Policy; Fiscal Policy; Time Consistency.

JEL codes: E31, E43, E62, E63

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1 Introduction

The Volcker disinflation of the late 1970s and early 1980s took place when the US government debt-to-GDP ratio was near its post-WWII low of 24% in Q3 of 1980.[1] Arguably, it was at this point that the US Fed re-asserted its independence. Since then, government debt levels in the US have risen significantly to 97% by Q1 2024. There has been a similar increase in government debt across the Eurozone as individual economies had to meet the Maastricht criteria, including debt-to-GDP ratios below 60% prior to the creation of the Euro in 1999, which have since risen, on average, to 91.6%.[2] While in the UK, net debt has risen from 36.6% of GDP at the time of the granting of operational independence to the Bank of England in 1997 to 98.5% by Q1 2024.[3] Similarly, in Japan the gross debt-to-GDP ratio was 80.5% at the time of central bank independence in 1998 and 255% by Q1 of 2024.[4] [5] Real interest rates have fallen over the same time period, such that the rise in debt is not obviously due to a deterioration in debt service costs following central bank independence. In this paper, we explore the interactions between an inflation averse independent central bank and a government setting fiscal policy.

We show that granting a central bank independence would reduce inflation initially but may also encourage the government to accumulate debt relative to the level that would have been chosen were monetary policy set by the government alongside fiscal policy. Eventually, this higher debt level will worsen inflation relative to what it would be without independence, despite the operational independence and inflation aversion of the central bank. The lower real interest rates observed in recent decades may further encourage the accumulation of debt and can explain why governments have allowed debt to rise by so much. However, were the trend to reverse and interest rates to return to 'normal' then this will lead to a significant and sustained rise in inflation, despite the central bank's independence, until the fiscal authorities have reduced the high debt levels. This could be the situation we are entering into now.

It is important to stress that this is not a story of fiscal dominance or of a 'game of chicken' between the central bank and the government. Instead, we have an independent central bank that sets interest rates to maximize a weighted average of social welfare and additional costs associated with its aversion to inflation i.e. we have an operationally independent conservative central bank. In doing so, it has no fiscal objectives and takes the government's fiscal choices as given. Similarly, fiscal authorities make their tax and spending decisions in order to maximize social welfare, taking monetary policy as given.

Instead, the key economic mechanism underpinning this result is the debt stabilization bias discussed in Leeper and Leith (2016) and Leeper et al. (2021). With a single policy maker, the existence of a stock of nominal debt, which requires costly tax increases or spending cuts to service, gives rise to an inflation bias problem. The policymaker faces the temptation to induce inflation surprises to reduce the real value of government debt.

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[1]This is the market value of government debt in the hands of the public taken from the Kansas Fed and scaled by GDP - https://fred.stlouisfed.org/series/FYGFGDQ188S

[2]Of the 11 original Eurozone members only Ireland, Luxembourg, and the Netherlands met the 60% debt target by 2022. Data Source: Eurostat Online data code GOV_10DD_EDPT1

[3]Source: Office of Budget Responsibility Public Finances Databank, https://obr.uk/data/

[4]Source: IMF Datamapper, https://www.imf.org/external/datamapper/profile/JPN

[5]An exception to this rule is Canada which did experience a steep rise in debt following independence for the Bank of Canada in 1991, rising from a pre-independence ratio of 73.7%, peaking at over 100% of GDP in 1996 but this was then reversed until rising again following the financial crisis to its current level of 107% - see https://www.imf.org/external/datamapper/GG$_DEBT_GDP@GDD/USA/CAN$.

Economic agents anticipate this, and we end up worsening the inflation bias problem relative to the case of inefficiencies due to monopolistic competition and distortionary taxation alone. However, by reducing the debt level, the policymaker can reduce the inflationary bias associated with government debt. This means that, following shocks, the policy maker will return to the steady-state which balances the costs of the inflation bias against the costs of fiscal consolidation. In contrast, the Ramsey policy under commitment would allow debt to absorb shocks permanently by following a tax smoothing policy. We labeled this sub-optimal return of debt to steady-state the 'debt stabilization bias'.

When the central bank is independent, their inflation aversion means that a given level of debt implies a smaller inflation bias than it otherwise would. Therefore, the immediate benefit of central bank independence is a reduction in inflation. However, the fact that inflation is lower reduces the government's desire to reduce debt, and equilibrium debt levels will rise relative to the case of a single policymaker. As debt levels rise, the inflation bias associated with debt increases, and we end up in a situation where central bank independence reduces inflation initially but subsequently discourages fiscal discipline. As a result, debt levels rise until inflation is higher than it was pre-independence. This does not undermine the desirability of central bank independence - even though inflation and debt levels rise in the long run, reversing central bank independence at that point would dramatically increase inflation even further due to the high level of indebtedness. Welfare is, therefore, always improved by central bank independence across all possible levels of debt and shock. However, we also show that welfare can be improved further by allowing the fiscal authority to adopt a (small) degree of debt aversion beyond that implied by social welfare.

This mechanism depends crucially on the private sector's desire to hold government debt. In recent decades, but especially during the financial crisis and subsequent pandemic, bond yields have fallen, sometimes turning negative. Various explanations have been offered for this - see Blanchard (2019) for a discussion - including a flight to safety reducing the yields on safe assets, aging populations increasing the desire to save, and increased inequality leading to an accumulation of savings by the rich. Regardless of the cause, the reduction in debt interest costs weakens the debt stabilization bias and allows debt levels to increase substantially with only a modest increase in inflation. However, when debt yields return to normal, the debt stabilization bias reasserts itself. The high levels of debt fuel inflation and prompt the fiscal authorities to reduce debt aggressively. Recent increases in bond yields and inflation are consistent with this phenomenon.

1.1 Literature Review

While there is a large literature examining the interactions between monetary and fiscal policies (see the discussion in Leeper and Leith (2016)), the number of papers exploring strategic interactions between a fiscal authority and an independent central bank is much more limited.[6] In the context of sticky-price New Keynesian economies like ours, notable

[6] There is also a literature allowing the central bank to have some degree of commitment, which the fiscal authority lacks - see, for example, Eggertsson (2013), Gnocchi (2013), de Beauffort (2024) and Camous and Matveev (2022). This may take the form of adopting simple rules or, for example, the central bank committing to ignore the repercussions of its actions on government debt, even though debt is a relevant state variable when the private sector is forming inflationary expectations. In our analysis, the only commitment is to the inflation conservatism possessed by the central bank - all policy decisions

examples include, Dixit and Lambertini (2003), Adam and Billi (2008, 2014) and Schreger et al. (2023). Of these papers, only Schreger et al. (2023) include a meaningful role for government debt, although they consider a two-period economy that cannot assess the longer-term evolution of debt dynamics and inflation. Our stochastic model includes sticky prices, long-term debt, inflation conservatism and policy-maker myopia. Outside of the New Keynesian literature, flexible price economies subject to monetary frictions, are considered in Alvarez et al. (2004), Chari and Kehoe (2007), Niemann (2011), Niemann et al. (2013) and Aguiar et al. (2015) and, using a Lagos-Wright monetary search model, Martin (2015).[7]

In terms of the results obtained, these papers tend to find that monetary conservatism is welfare improving, although Niemann (2011) suggests that complete conservatism can lead to an excessive tolerance of high debt. Our contribution lies in considering strategic interactions between an inflation-averse independent central bank that controls interest rates and a fiscal authority which issues debt, levies a distortionary income tax, and chooses the level of government consumption. The policymakers may potentially be myopic, and government debt is long-term (which affects the debt stabilization bias discussed in Leeper et al. (2021)). Prices are sticky, which ensures we capture the transmission mechanisms of both monetary and fiscal policies typically considered in empirical work on money and fiscal interactions - see, for example, Bianchi (2012), Bianchi and Melosi (2017) and Chen et al. (2022). We then assess how inflation aversion, myopia, and strategic interactions affect both inflation and debt accumulation over time.

Beyond the literature on strategic interactions, there are also a number of papers building on the 'unpleasant monetarist arithmetic' of Sargent and Wallace (1981). They consider a 'game of chicken' between the two policymakers where it is unclear whether it is the fiscal or monetary authority who will ultimately act to stabilize government debt. Contemporary extensions of this kind of analysis include Davig et al. (2010) and Bianchi and Melosi (2019, 2022) where current inflation may be generated by a failure of the fiscal authority to make clear that they will eventually use their fiscal instruments to ensure fiscal solvency. Bianchi et al. (2022) also proposes that coordinated action by the monetary and fiscal authorities may enable a period of controlled inflation to successfully reduce the burden of government debt. The current paper is not concerned with the issue of Fiscal vs Monetary dominance in this sense, and instead, the monetary and fiscal authorities interact strategically period-by-period, seeking to achieve their objectives taking the policies of the other as given. At no point does the monetary authority stop attempting to fulfill its anti-inflation mandate in order to bail out the government.

Roadmap. The paper proceeds as follows. We describe the benchmark model in section 2. The non-linear time-consistent problem with strategically interacting monetary and fiscal authorities is described in section 3. We develop a simple model two-period model to provide intuition for results in section 4 before undertaking a numerical analysis of the equilibrium implied by the full model in section 5, exploring the implications of variations in policymaker myopia and central bank conservatism. In extensions, we examine the implications of an unexpected crisis which includes a sustained reduction in the natural rate of interest in section 6. We conclude in Section 7.

are time-consistent thereafter.

[7]A number of papers also explore how fiscal policy can influence inflation expectations when monetary policy is at the ZLB - see, for example, Eggertsson (2006), Burgert and Schmidt (2014) and de Beauffort (2023). Due to the inflation bias generated by government debt, the ZLB is not an issue in our model.

2 The Model

Our model is a standard New Keynesian model but augmented to include the government's budget constraint where government spending is financed by distortionary taxation and/or borrowing. This basic setup is similar to that in Leeper et al. (2021), which in turn follows Benigno and Woodford (2003), and Schmitt-Grohe and Uribe (2004) but with some differences. Firstly, we allow the government to optimally vary government spending in the face of shocks, rather than simply treating government spending as an exogenous flow which must be financed. This is a necessary modification to answer questions like the relative effectiveness of government spending cuts and tax increases in debt stabilization. Secondly, our nominal debt is not of single-period maturity but consists of a portfolio of bonds of mixed maturities. In reality, most countries issue long-term nominal debt in overwhelming proportions of total debt. This is an important consideration in highly indebted economies since even modest surprise changes in inflation and interest rates can have substantial effects on the market value of debt, and hence become a sizeable source of fiscal revenue.[8] This fact suggests that the maturity structure of debt is an essential element in characterizing the jointly optimal monetary and fiscal policies.

We have two policymakers - an independent central bank which sets short-term interest rates in order to achieve its objectives which are given by social welfare less a cost in deviating from a delegated inflation target. In other words, they are an inflation-averse operationally independent central bank.[9] We also allow for the possibility of 'myopia' on the part of the monetary authority, reflecting the fact that monetary policy is typically concerned with the medium-term stabilization of inflation. The central bank has no fiscal objectives and takes the government's setting of its policy instruments as given when setting monetary policy. Fiscal policy, in the form of an income tax and public consumption, is under the control of the government. The government's objectives may coincide with social welfare, or they may be more myopic - a short-hand way of describing a 'populist' policymaker who does not fully take account of the long-run consequences of their actions.[10] Each policy maker acts taking the other's policy instruments as given, and the equilibrium outcome outlined in the section 3 represents the Nash equilibrium of a strategic game between the two players with simultaneous moves.

We begin by deriving the model before turning to the detailed specification of the policy problem in the following section.

2.1 Households

There are a continuum of households of size one. Households appreciate private consumption as well as the provision of public goods and dislike supplying labor. We shall assume complete asset markets such that, through risk sharing, they will face the same budget constraint. As a result, the typical household will seek to maximize the following objective function,

[8] See Hall and Sargent (2011) and Sims (2013) for the empirical findings on the contribution of this kind of fiscal financing to the decline in the U.S. debt-GDP ratio from 1945 to 1974.

[9] They do not care only about inflation since that would make them, in the words of former Bank of England Governor Mervyn King an "inflation nutter" and would induce them to change interest rate policy erratically to achieve the inflation target in each period.

[10] In Section 5.4, we also allow the fiscal authority to adopt a degree of debt aversion beyond that implied by social welfare in a manner akin to that of the inflation aversion of the central bank. We find this can help support the central bank's anti-inflation policy.

$$E_0 \sum_{t=0}^{\infty} (\prod_{i=-1}^{t-1} \beta_i) U\left(C_t, N_t, G_t\right) \tag{1}$$

where C, G, and N are a consumption aggregate, a public goods aggregate, and labor supply, respectively. We have allowed the households' discount factor to vary over time. For most of the analysis, we shall consider the discount factor to be fixed, but in section 6 we shall introduce variation in the natural rate of interest through the device of exogenous Markov switching in households' regime-dependent steady-state discount factors. For the moment, it is important to note that the timing of the discount factor implies that β_t is the discount factor used to discount next period $t+1$ utility at time t. Therefore, households know their discount factor as it applies to next period's utility, but it may change beyond that, and that needs to be factored into their expectations.

The consumption aggregate is defined as

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{2}$$

where j denotes the good's type or variety and $\epsilon_t > 1$ is the elasticity of substitution between varieties. $\ln(\epsilon_t)$ is AR(1) such that $\ln(\epsilon_t) = (1+\rho_\epsilon)\ln(\epsilon) + \rho_\epsilon \ln(\epsilon_{t-1}) + e_{\epsilon t}$, with $0 \le \rho_\epsilon < 1$ and $e_{\epsilon t} \overset{i.i.d}{\sim} N(0, \sigma_\epsilon^2)$. This serves as a device to introduce mark-up shocks to the model. The public goods aggregate takes the same form

$$G_t = \left(\int_0^1 G_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{3}$$

The budget constraint at time t is given by

$$\int_0^1 P_t(j)C_t(j)dj + P_t^S B_t^S + P_t^M B_t^M \le \Xi_t + (1+\rho P_t^M)B_{t-1}^M + B_{t-1}^S + W_t N_t (1-\tau_t) + P_t Tr_t$$

where $P_t(j)$ is the price of variety j , Ξ is the representative household's share of profits in the imperfectly competitive firms, $P_t Tr_t$ are the nominal value of lump-sum transfers from the government, W_t are wages, and τ_t is an wage income tax rate.[11] Households hold two basic forms of government bonds. The first is the familiar one period debt, B_t^S which has the price equal to the inverse of the gross nominal interest rate, $P_t^S = R_t^{-1}$. The second type of bond is actually a portfolio of many bonds which, following Woodford (2001) pay a declining coupon of ρ^j dollars $j+1$ periods after they were issued where $0 < \rho \le \beta_j^{-1}$,$j = H, L$. The steady-state duration of the bond $(1-\beta_j\rho)^{-1}$, which allows us to vary ρ as a means of changing the implicit maturity structure of government debt. By using such a simple structure, we need only price a single bond, since any existing bond issued j periods ago is worth ρ^j new bonds. In the special case where $\rho = 1$ these bonds become infinitely lived consols.[12]

[11] Since fiscal policy is an important element of this paper, we do not assume any kind of lump-sum-tax-financed subsidy to offset the distortion arising from monopolistic competition, which is a not uncommon assumption in the optimal fiscal and monetary policy literature using New Keynesian models. Thus, the steady state of the model economy is not efficient. Moreover, it cannot be determined outside of the policy problem itself, requiring the use of global solution methods.

[12] This way of modeling long-term debt is quite elegant since it allows us to study long-duration bonds

Households must first decide how to allocate a given level of expenditure across the various goods that are available. They do so by adjusting the share of a particular good in their consumption bundle to exploit any relative price differences - this minimizes the costs of consumption. Optimization of expenditure for any individual good implies the demand function given below,

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} C_t$$

where we have price indices given by

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_t} dj\right)^{\frac{1}{1-\epsilon_t}}$$

The budget constraint can, therefore, be rewritten as

$$P_t^S B_t^S + P_t^M B_t^M \leq \Xi_t + (1 + \rho P_t^M) B_{t-1}^M + B_{t-1}^S + W_t N_t (1 - \tau_t) - P_t C_t + P_t T r_t \tag{4}$$

where $\int_0^1 P_t(j) C_t(j) dj = P_t C_t$. P_t is the current price level. The constraint says that total financial wealth in period t can be worth no more than the value of financial wealth brought into the period plus nonfinancial income during the period net of taxes and the value of consumption spending.

Throughout the analysis, one-period government bonds B_t^S will be in zero net supply with beginning-of-period price P_t^S, while the general portfolio of government bond B_t^M is in non-zero net supply with beginning-of-period price P_t^M. Higher ρ raises the maturity of the bond portfolio. In the special case, where $\rho = 0$, the debt portfolio collapses to one-period debt.

Similarly, the allocation of government spending across goods is determined by minimizing total costs, $\int_0^1 P_t(j) G_t(j) dj$. Given the form of the basket of public goods, this implies,

$$G_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} G_t$$

2.1.1 Households' Intertemporal Consumption Problem

The first of the household's intertemporal problems involves allocating consumption expenditure across time. To facilitate numerical implementation, we assume that (1) takes the specific form,

$$E_0 \sum_{t=0}^{\infty} (\prod_{i=-1}^{t-1} \beta_i) \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) \tag{5}$$

We can then maximize utility subject to the budget constraint (4) to obtain the optimal allocation of consumption across time based on the pricing of one period bonds,

$$\beta_t R_t E_t \left\{ \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \left(\frac{P_t}{P_{t+1}}\right) \right\} = 1 \tag{6}$$

without increasing the dimensionality of the state space, and it is commonly adopted in the literature (e.g., Eusepi and Preston, 2012; Chen et al., 2012).

and the declining payoff consols,

$$\beta_t E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) \left(1 + \rho P_{t+1}^M\right) \right\} = P_t^M \tag{7}$$

Combining (6) and (7) yields the no-arbitrage condition between one-period and long-term bonds,

$$P_t^M = E_t \left[P_t^S \left(1 + \rho P_{t+1}^M\right)\right] \tag{8}$$

where $P_t^S = R_t^{-1}$. Notice that when these reduce to single period bonds, $\rho = 0$, the price of these bonds is also given by $P_t^M = R_t^{-1}$. However, outside of this special case, the longer-term bonds introduce the term structure of interest rates to the model. It is convenient to define the stochastic discount factor (for nominal payoffs) for later use,

$$\beta_t \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) = Q_{t,t+1}$$

The second FOC relates to their labor supply decision and is given by,

$$(1 - \tau_t) \left(\frac{W_t}{P_t} \right) = N_t^{\varphi} C_t^{\sigma}$$

That is, the marginal rate of substitution between consumption and leisure equals the after-tax wage rate. Besides these FOCs, necessary and sufficient conditions for household optimization also require the household's budget constraints to bind with equality. In addition, there is an associated no-Ponzi-game condition derived as follows. Define household wealth brought into period t as,

$$D_t = (1 + \rho P_t^M) B_{t-1}^M + B_{t-1}^S$$

the no-Ponzi-game condition can be written as,

$$\lim_{T \to \infty} E_t \left[\frac{1}{R_{t,T}} \frac{D_T}{P_T} \right] \geq 0 \tag{9}$$

where

$$R_{t,T} = \prod_{s=t}^{T-1} \left(\frac{1 + \rho P_{s+1}^M}{P_s^M} \frac{P_s}{P_{s+1}} \right)$$

for $T \geq 1$ and $R_{t,t} = 1$, also see Eusepi and Preston (2011). The no-Ponzi-game condition says that the present discounted value of household's real wealth at infinity is non-negative, that is, there is no overaccumulation of debt. In equilibrium, the condition holds with equality.

2.2 Firms

The production function is linear, so for firm j

$$Y_t(j) = N_t(j) \tag{10}$$

The real marginal costs of production is defined as $mc_t = W_t/(P_t)$. The demand curve they face is given by,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} Y_t$$

where $Y_t = \left[\int_0^1 Y_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj\right]^{\frac{\epsilon_t}{\epsilon_t - 1}}$. Firms are also subject to quadratic adjustment costs in changing prices, as in Rotemberg (1982).

We define the Rotemberg price adjustment costs for a monopolistic firm j as,

$$v_t(j) = \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t \tag{11}$$

where $\phi \geq 0$ measures the degree of nominal price rigidity. The adjustment cost, which accounts for the negative effects of price changes on the customer–firm relationship, increases in magnitude with the size of the price change and with the overall scale of economic activity Y_t.

The problem facing firm j is to maximize the discounted value of profits,

$$\max_{P_t(j)} E_t \sum_{z=0}^{\infty} Q_{t,t+z} \Xi_{t+z}(j)$$

where profits are defined as,

$$\begin{aligned}
\Xi_t(j) &= P_t(j)Y_t(j) - mc_t Y_t(j) P_t - \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t \\
&= P_t(j)^{1-\epsilon_t} P_t^{\epsilon_t} Y_t - mc_t P_t(j)^{-\epsilon_t} P_t^{1+\epsilon_t} Y_t - \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t
\end{aligned}$$

So that, in a symmetric equilibrium where $P_t(j) = P_t$ the first order conditions are given by,

$$\begin{aligned}
0 = {} & (1 - \epsilon_t) + \epsilon_t mc_t - \phi \Pi_t (\Pi_t - 1) \\
& + \phi \beta_t E_t \left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} (\Pi_{t+1} - 1)\right]
\end{aligned} \tag{12}$$

which is Rotemberg's version of the New Keynesian Phillips curve relationship.

Goods market clearing requires, for each good j,

$$Y_t(j) = C_t(j) + G_t(j) + v_t(j)$$

which allows us to write,

$$Y_t = C_t + G_t + v_t$$

with $v_t = \int_0^1 v_t(j)\, dj$. In a symmetrical equilibrium,

$$Y_t\left[1 - \frac{\phi}{2}(\Pi_t - 1)^2\right] = C_t + G_t$$

There is also market clearing in the bond market, where we assume that one period bonds are in zero net supply, $B_t^S = 0$ and the remaining portfolio of longer-term bonds evolves according to the government's budget constraint which we will now describe.

2.3 Government Budget Constraint

There are two policy makers - monetary and fiscal. The monetary authority controls the nominal interest rate on short-term nominal bonds. The fiscal authority chooses the level of government consumption, labor income taxes, and debt policy. In addition to government consumption, government expenditures also consist of exogenously determined transfers, $P_t Tr_t$, and the interest payments on outstanding debt.

Government expenditures are financed by levying labor income taxes at the rate τ_t, and by issuing one-period, risk-free (non-contingent), nominal obligations B_t^S, and long term bonds B_t^M. The government's sequential budget constraint is then, assuming the one-period bond is in zero net supply, $B_t^S = 0$, given by,

$$P_t^M B_t^M = (1 + \rho P_t^M) B_{t-1}^M - W_t N_t \tau_t + P_t G_t + P_t Tr_t \tag{13}$$

Note that $(1 + \rho P_t^M) B_{t-1}^M$ is outstanding government liabilities in period t. Distortionary taxation and spending adjustments are required to service government debt as well as stabilize the economy. Rewriting in real terms

$$P_t^M b_t = (1 + \rho P_t^M) \frac{b_{t-1}}{\Pi_t} - \frac{W_t}{P_t} N_t \tau_t + G_t + Tr_t \tag{14}$$

where $b_t \equiv B_t^M / P_t$ represents the ratio of the number of nominal bonds to the price level, a ratio that will be stationary despite the economy being subject to an endogenously determined rate of inflation.

Given that government debt is nominal and prices are sticky, monetary policy decisions affect the government budget through various channels: first, monetary policy affects debt service costs; second, in a sticky price economy, monetary policy has real effects which impact the tax base and, for a given tax rate, the government's primary surplus.

Monetary policy's impact on debt service costs depends on the maturity structure of the debt. In (14), the amount of outstanding real government debt is $P_t^M b_t$, and the period real return on holding government debt is $(1 + \rho P_t^M) / \left(\Pi_t P_{t-1}^M\right)$. If $\rho = 0$, government debt b_t is reduced into one-period debt, and then the only way to adjust the real return on bonds *ex post* is through inflation in the current period Π_t. Large price fluctuations can be very costly in the presence of nominal rigidities. However, if government debt has a longer maturity, $0 < \rho < 1$, adjustments in the *ex post* real return can be engineered via changes in the bond price P_t^M, which depends on inflation in future periods. This means that changes in the real debt return can be produced by a small but sustained inflation, which is less costly than equivalent large fluctuations in inflation. As a result, long-term debt can help the policy maker achieve the desired adjustment in the *ex post* real return at a smaller cost.

That completes the description of our model, which contains the usual resource constraint, consumption Euler equation, and New Keynesian Phillips curve as well as the government's budget constraint and the bond pricing equation for longer-term bonds. These equations and the debt-dependent steady state are described in Appendix B.1.

3 Simultaneous Moves Nash Equilibrium

In this section, we allow the two authorities to possess different objective functions and specify the time-consistent policy problem, which considers the resultant strategic interactions. We allow the objective functions to differ from social welfare in two ways. Firstly, the monetary authority may have an additional dislike of inflation beyond that contained in the social welfare function. Secondly, we also allow either or both policymakers to discount the future at a different rate from households.

Monetary conservatism is formulated as in Adam and Billi (2008). Therefore, we consider the central bank's flow utility function to be given by,

$$U_t^M = (1-\alpha_\pi)\left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\chi G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi}\right) - \frac{\alpha_\pi}{2}\left(\Pi_t - 1\right)^2 \tag{15}$$

which means they suffer an additional utility loss if net inflation is non-zero. $0 \leq \alpha_\pi \leq 1$ measures the degree of monetary conservatism. In particular, $\alpha_\pi = 1$ means that the policymaker cares about inflation only. It is important to note that even with $\alpha_\pi = 0$, inflation carries a welfare cost in a sticky-price environment, and this will be reflected in policymaker behavior, $\alpha_\pi > 0$ is a measure of inflation aversion beyond that implied by the social costs of inflation. The fiscal authority's flow objective, U_t^F, takes the same form but with $\alpha_\pi = 0$.

We also allow the discount factors of each policy maker to differ, such that their respective objectives are given by,

$$E_0 \sum_{t=0}^{\infty} (\beta^i)^t U_t^i \text{ where } i = M, F. \tag{16}$$

That the fiscal authorities may have a shorter time horizon than society, $\beta^F \leq \beta$ is a common assumption motivated by various political economy frictions that can give rise to a short-term outlook (see Alesina and Passalacqua (2016) for a discussion). The relevant discount rate for the independent central bank is less obvious. Some authors simply adopt the discounting of households for the central bank (see, for example, Woodford (2003)). However, central bank governors and interest rate-setting committee members typically have fixed-term contracts, which may influence their time horizon. Moreover, their discussion of the future often focuses on the lags in the monetary policy transmission mechanism such that monetary policy is set so as to achieve its target in the medium term.[13] There is an often unspoken assumption that nothing prevents the central bank from achieving its target other than these short-term constraints, and central banks will typically forecast inflation returning to target over such time horizons. As discussed in Leeper and Leith (2016), this implicitly means that a strong set of conditions that allow monetary policy alone to determine inflation outcomes must be fulfilled, including that fiscal policy uses lump-sum taxes to implement a passive fiscal rule. As a result, we also allow for the possibility that the central bank is myopic, $\beta^M \leq \beta$.

Given this conflict in objectives, we have scope for strategic interactions between the two policymakers. The solution concept we consider is the Nash equilibrium to a simultaneous moves game where each policy maker sets their current policy instrument(s)

[13] For example, former Bank of England Governor Mervyn King concludes, " ... it seems sensible for the central bank to target inflation something like two years ahead." (King, 2000).

given those set by the other player. The resultant equilibrium will be time-consistent, and neither policymaker has the ability to commit to future policies. However, they can influence the future through the impact of their current actions on endogenous state variables, specifically government debt. We capture this by defining the auxiliary functions,

$$M(b_t, \beta_{t+1}, \epsilon_{t+1}) = (C_{t+1})^{-\sigma} Y_{t+1}\Pi_{t+1} (\Pi_{t+1} - 1)$$

$$L(b_t, \beta_{t+1}, \epsilon_{t+1}) = (C_{t+1})^{-\sigma}(\Pi_{t+1})^{-1}(1 + \rho P^M_{t+1})$$

which can then replace the relevant expectations terms in the constraints facing each policymaker.

3.1 The Fiscal Authority's Problem

We begin by considering the fiscal authority's problem where it maximizes the following Lagrangian, taking monetary policy as given (Note we are implicitly treating consumption as the monetary authority's instrument since they can costlessly adjust interest rates to achieve any value for consumption they wish, given the values of all other variables.),

$$\begin{aligned}
\mathcal{L}^f = &\left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\chi G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} + \beta^F E_t[V^f_{t+1}(b_t, \beta_{t+1}, \epsilon_{t+1})] \right\} \\
&+ \lambda^f_{1t} \left[Y_t \left(1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right) - C_t - G_t \right] \\
&+ \lambda^f_{2t} \left[\begin{array}{c} (1-\epsilon_t) + \epsilon_t(1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} - \phi \Pi_t (\Pi_t - 1) \\ +\phi \beta_t C_t^{\sigma} Y_t^{-1} E_t [M(b_t, \beta_{t+1}, \epsilon_{t+1})] \end{array} \right] \\
&+ \lambda^f_{3t} \left[\begin{array}{c} \beta_t b_t C_t^{\sigma} E_t [L(b_t, \beta_{t+1}, \epsilon_{t+1})] - \frac{b_{t-1}}{\Pi_t} (1 + \rho \beta_t C_t^{\sigma} E_t [L(b_t, \beta_{t+1}, \epsilon_{t+1})]) \\ + \left(\frac{\tau_t}{1-\tau_t} \right) Y_t^{1+\varphi} C_t^{\sigma} - G_t - Tr_t \end{array} \right]
\end{aligned}$$

We can write the first-order conditions (FOCs) for the policy problem as follows:

Government spending,

$$\chi G_t^{-\sigma_g} - \lambda^f_{1t} - \lambda^f_{3t} = 0 \tag{17}$$

which says that the government matches the marginal utility gain from higher government spending against the tightening of the resource constraint ($\lambda^f_{1t} \geq 0$), and government budget constraint ($\lambda^f_{3t} \geq 0$).

Output,

$$\begin{aligned}
-Y_t^{\varphi} + \lambda^f_{1t} &\left[1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right] \\
+\lambda^f_{2t} &\left[\epsilon_t \varphi (1-\tau_t)^{-1} Y_t^{\varphi-1} C_t^{\sigma} - \phi \beta_t C_t^{\sigma} Y_t^{-2} E_t [M(b_t, \beta_{t+1}, \epsilon_{t+1})] \right] \\
+\lambda^f_{3t} &\left[(1+\varphi) Y_t^{\varphi} C_t^{\sigma} \left(\frac{\tau_t}{1-\tau_t} \right) \right] = 0
\end{aligned} \tag{18}$$

such that the marginal costs of higher output - the reduction in utility due to the need to increase labor supply and the fuelling of inflation this implies,($\lambda^f_{2t} \leq 0$) - are equated to the marginal gains from relaxing the resource constraint ($\lambda^f_{1t} \geq 0$) (facilitating enhanced private and/or public consumption) and the fiscal benefits of increasing the tax base,

($\lambda_{3t}^f \geq 0$).

Taxation,

$$\lambda_{2t}^f \left[\epsilon_t (1-\tau_t)^{-2} Y_t^{\varphi} C_t^{\sigma}\right] + \lambda_{3t}^f \left[Y_t^{1+\varphi} C_t^{\sigma} (1-\tau_t)^{-2}\right] = 0$$

simplifying,

$$\epsilon_t \lambda_{2t}^f + \lambda_{3t}^f Y_t = 0 \tag{19}$$

which is the marginal condition capturing the fact that a higher (distortionary) tax rate increases marginal costs and fuels inflation t ($\lambda_{2t}^f \leq 0$), while at the same time generating tax revenues which relaxes the government's budget constraint ($\lambda_{3t}^f \geq 0$);

Inflation,

$$\begin{aligned} &-\lambda_{1t}^f \left[Y_t \phi \left(\Pi_t - 1\right)\right] - \lambda_{2t}^f \left[\phi \left(2\Pi_t - 1\right)\right] \\ &+\lambda_{3t}^f \left[\frac{b_{t-1}}{\Pi_t^2} \left(1 + \rho \beta_t C_t^{\sigma} E_t \left[L(b_t, \beta_{t+1}, \epsilon_{t+1})\right]\right)\right] = 0 \end{aligned} \tag{20}$$

This condition exists since the monetary authority doesn't directly control inflation, implying that the fiscal authority also evaluates the inflationary consequences of its actions. The first line captures the conventional inflationary bias problem from the perspective of the fiscal authority. A higher inflation rate has direct resource costs ($\lambda_{1t}^f \geq 0$), but increases output through the NKPC, conditional on expectations at time t ($\lambda_{2t}^f \leq 0$). However, there is an additional benefit to inflation in that, again conditional on expectations, it deflates the real value of government debt, thereby relaxing the government's budget constraint ($\lambda_{3t}^f \geq 0$).

Government debt,

$$\beta^F E_t \left[\frac{\partial V_{t+1}^f (b_t, \beta_{t+1}, \epsilon_{t+1})}{\partial b_t}\right] + \lambda_{2t}^f \left[\phi \beta_t C_t^{\sigma} Y_t^{-1} E_t \left[M_1(b_t, \beta_{t+1}, \epsilon_{t+1})\right]\right]$$

$$+\beta_t \lambda_{3t}^f \left[C_t^{\sigma} E_t \left[L(b_t, \beta_{t+1}, \epsilon_{t+1})\right] + b_t C_t^{\sigma} E_t \left[L_1(b_t, \beta_{t+1}, \epsilon_{t+1})\right] - \rho \frac{b_{t-1}}{\Pi_t} C_t^{\sigma} E_t \left[L_1(b_t, \beta_{t+1}, \epsilon_{t+1})\right]\right] = 0$$

where

$$\begin{aligned} L_1(b_t, \beta_{t+1}, \epsilon_{t+1}) &\equiv \partial L(b_t, \beta_{t+1}, \epsilon_{t+1}) / \partial b_t \\ M_1(b_t, \beta_{t+1}, \epsilon_{t+1}) &\equiv \partial M(b_t, \beta_{t+1}, \epsilon_{t+1}) / \partial b_t \end{aligned}$$

Note that by the envelope theorem,

$$\frac{\partial V_t^f (b_{t-1}, \beta_t, \epsilon_t)}{\partial b_{t-1}} = -\frac{\lambda_{3t}^f}{\Pi_t} \left(1 + \rho \beta_t C_t^{\sigma} E_t \left[L(b_t, \beta_{t+1}, \epsilon_{t+1})\right]\right)$$

the FOC for taxation (19), and the definition of bond prices (7), we can write the FOC for government debt as,

$$\begin{aligned} &P_t^M \lambda_{3t}^f - \beta^F E_t \left[\frac{\lambda_{3t+1}^f}{\Pi_{t+1}} \left(1 + \rho P_{t+1}^M\right)\right] \\ &-\beta_t \lambda_{3t}^f C_t^{\sigma} \left[\phi \epsilon_t^{-1} E_t \left[M_1(b_t, \beta_{t+1}, \epsilon_{t+1})\right] - (b_t - \rho \frac{b_{t-1}}{\Pi_t}) E_t \left[L_1(b_t, \beta_{t+1}, \epsilon_{t+1})\right]\right] = 0 \end{aligned} \tag{21}$$

The first line of this expression describes how the fiscal authority trades off the current and future distortions associated with the government budget constraint. This element of the fiscal authority's FOC would also exist under commitment. The second line captures the debt stabilization bias, which does not exist under commitment.

In the absence of any fiscal myopia $\beta = \beta^F$, the first line would describe a policy of tax smoothing as, in Barro (1979). If the (risk-adjusted) real return to bonds equalled the rate of time preference of the household/government, then policy would ensure the distortions associated with the budget constraint are constant, and steady-state government debt would follow a random walk. In other words, following shocks, government debt will settle at a new steady state where, for example, in the case of a negative shock, the costs of undertaking a fiscal consolidation to reduce debt exactly balance the costs of servicing that debt. When the government is myopic, $\beta > \beta^F$ they would deviate from tax smoothing by allowing debt (and the distortions associated with debt, captured by λ_{3t}^f), to rise even when the real return to debt equalled the households' rate of time preference. [14]

Under the time-consistent policy, however, the fiscal authority's desired path of fiscal distortions also depends upon the influence debt has on expectations via the derivatives of the auxiliary functions on the second line of (21). The first term within the second set of square brackets, $M_1(b_t, \beta_{t+1}, \epsilon_{t+1}) > 0$, captures the increase in inflation expectations associated with an increase in debt. Since inflation is costly in our economy, this encourages the fiscal authority to further deviate from tax smoothing and reduce debt. The second term, $L_1(b_t, \beta_{t+1}, \epsilon_{t+1}) < 0$, instead measures the reduction in expected bond yields associated with increases in government debt. This also serves to discourage the accumulation of debt. Falling debt levels will imply rising bond prices, which make it cheaper to issue debt but more costly to repay that debt (when it is of longer maturity). Therefore, the shorter the maturity of debt, the greater incentive the government has to repay debt due to this mechanism.

In summary, a non-myopic government would like to smooth taxes and hold debt at a new sustainable level following shocks. However, when they cannot commit, they wish to deviate from tax smoothing and return debt to a pre-shock steady state due to the negative impact debt has on both inflation expectations and the costs of bond issuance (particularly for shorter maturity debt). This tendency may be offset to the extent that the fiscal authority is myopic.

This describes the incentives facing the fiscal authority, which takes monetary policy as given. In order to describe the Nash equilibrium of the policy game between the two policymakers, we also need to consider the central bank's policy problem and how monetary policy interacts with the fiscal policy we have just described.

3.2 Monetary Authority's Problem

We now consider the monetary authority's policy problem where it maximizes the following Lagrangian taking the fiscal instruments, τ_t and G_t as given.

[14]Note that the *ex ante* real return on government bonds, $E_t\left[\frac{(1+\rho P_{t+1}^M)}{\Pi_{t+1} P_t^M}\right]$, will vary depending on the monetary policy set by the central bank, thereby tilting the intertemporal tax smoothing trade-offs faced by the fiscal authority. This will be an additional channel through which the monetary authority affects the policy choices of the fiscal authority.

$$\mathcal{L}^m = \left\{ (1-\alpha_\pi)\left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\chi G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} \right) - \frac{\alpha_\pi}{2}(\Pi_t - 1)^2 + \beta^M E_t[V_{t+1}^m(b_t, \beta_{t+1}, \epsilon_{t+1})] \right\}$$
$$+ \lambda_{1t}^m \left[Y_t \left(1 - \frac{\phi}{2}(\Pi_t - 1)^2\right) - C_t - G_t \right]$$
$$+ \lambda_{2t}^m \left[\begin{array}{c} (1-\epsilon_t) + \epsilon_t (1-\tau_t)^{-1} Y_t^\varphi C_t^\sigma - \phi \Pi_t (\Pi_t - 1) \\ +\phi \beta_t C_t^\sigma Y_t^{-1} E_t [M(b_t, \beta_{t+1}, \epsilon_{t+1})] \end{array} \right]$$
$$+ \lambda_{3t}^m \left[\begin{array}{c} \beta_t b_t C_t^\sigma E_t [L(b_t, \beta_{t+1}, \epsilon_{t+1})] - \frac{b_{t-1}}{\Pi_t}(1 + \rho \beta_t C_t^\sigma E_t [L(b_t, \beta_{t+1}, \epsilon_{t+1})]) \\ + \left(\frac{\tau_t}{1-\tau_t}\right)(Y_t)^{1+\varphi} C_t^\sigma - G_t \end{array} \right]$$

The monetary authority's first-order condition for output is in the same form as that for the fiscal authority except that the first term is weighted by $(1-\alpha_\pi)$ and the Lagrange multipliers are those of the central bank, $\lambda_{it}^m, i = 1, 2, 3$. The re-weighting of the first term constitutes a reduced concern for the real economy on the part of the monetary authority which will be reflected in the values of its Lagrange multipliers. The FOC for consumption is given by,

$$(1-\alpha_\pi) C_t^{-\sigma} - \lambda_{1t}^m + \lambda_{2t}^m \left[\sigma \epsilon_t (1-\tau_t)^{-1} Y_t^\varphi C_t^{\sigma-1} + \sigma \phi \beta_t C_t^{\sigma-1} Y_t^{-1} E_t [M(b_t, \beta_{t+1}, \epsilon_{t+1})] \right]$$
$$+\lambda_{3t}^m \left[\begin{array}{c} \sigma \beta_t b_t C_t^{\sigma-1} E_t [L(b_t, \beta_{t+1}, \epsilon_{t+1})] - \rho \sigma \beta_t \frac{b_{t-1}}{\Pi_t} C_t^{\sigma-1} E_t [L(b_t, \beta_{t+1}, \epsilon_{t+1})] \\ +\sigma \left(\frac{\tau_t}{1-\tau_t}\right)(Y_t)^{1+\varphi} C_t^{\sigma-1} \end{array} \right] = 0 \qquad (22)$$

Here, the marginal conditions equate the marginal utility gain from consumption plus the relaxation of the government's budget constraint by reducing real interest rates, cet. par. ($\lambda_{3t}^m \geq 0$) against the tightening of the resource constraint ($\lambda_{1t}^m \geq 0$) and the worsening of the output-inflation trade-off at time t ($\lambda_{2t}^m \leq 0$).

The increased concern for inflation stabilization can then be seen in the FOC for inflation,

$$-\alpha_\pi (\Pi_t - 1) - \lambda_{1t}^m [Y_t \phi (\Pi_t - 1)] - \lambda_{2t}^m [\phi (2\Pi_t - 1)]$$
$$+\lambda_{3t}^m \left[\frac{b_{t-1}}{\Pi_t^2}(1 + \rho \beta_t C_t^\sigma E_t [L(b_t, \beta_{t+1}, \epsilon_{t+1})]) \right] = 0 \qquad (23)$$

which says that a higher inflation rate tightens the resource constraint ($\lambda_{1t}^m \geq 0$), has positive effects on the inflation-output trade-off at time t ($\lambda_{2t}^m \leq 0$), and relaxes the government budget constraint ($\lambda_{3t}^m \geq 0$) and where the first term captures the additional desire to reduce inflation when the central bank has gained conservatism, $\alpha_\pi > 0$. Therefore, the inflation bias balances the costs of inflation against the potential gains from surprise inflation in boosting output and reducing the real value of government debt. The additional inflation aversion tilts that balance in favor of lower inflation, *cet. par.*. This, in turn, implies that a given level of debt will be associated with a lower rate of inflation, reducing the fiscal authority's desire to reduce debt in line with condition, (21).

While the central bank takes fiscal policy as given and has no target for government debt levels or any other fiscal variables, they do recognize that their monetary policy actions impact government debt dynamics. The FOC for government debt is, therefore, given by,

$$\beta^M E_t \left[\frac{\partial V^m_{t+1}(b_t, \beta_{t+1}, \epsilon_{t+1})}{\partial b_t} \right] + \lambda^m_{2t} \left[\phi \beta_t C^\sigma_t Y^{-1}_t E_t \left[M_1(b_t, \beta_{t+1}, \epsilon_{t+1}) \right] \right]$$

$$+\beta_t \lambda^m_{3t} \left[C^\sigma_t E_t \left[L(b_t, \beta_{t+1}, \epsilon_{t+1}) \right] + b_t C^\sigma_t E_t \left[L_1(b_t, \beta_{t+1}, \epsilon_{t+1}) \right] - \rho \frac{b_{t-1}}{\Pi_t} C^\sigma_t E_t \left[L_1(b_t, \beta_{t+1}, \epsilon_{t+1}) \right] \right] = 0$$

As in the case of the fiscal authority, we can use the envelope theorem and the definition of bond prices (7), to write the FOC for government debt as,

$$P^M_t \lambda^m_{3t} - \beta^M E_t \left[\frac{\lambda^m_{3t+1}}{\Pi_{t+1}} \left(1 + \rho P^M_{t+1}\right) \right] + \lambda^m_{2t} \phi \beta_t C^\sigma_t Y^{-1}_t E_t \left[M_1(b_t, \beta_{t+1}, \epsilon_{t+1}) \right]$$

$$+\beta_t \lambda^m_{3t} C^\sigma_t (b_t - \rho \frac{b_{t-1}}{\Pi_t}) E_t \left[L_1(b_t, \beta_{t+1}, \epsilon_{t+1}) \right] = 0 \tag{24}$$

The monetary authority's FOC for debt has a similar form to that of the fiscal authority. However, given the fiscal authority's control of over the tax rate, we cannot interpret the first two terms of their condition as following the familiar logic of tax smoothing. Instead, in the case of the monetary authority, taking tax rates and government spending as given, the condition describes how the central bank will set monetary policy (through its impact on bond yields, where the ex ante real return on bonds is given by, $E_t \left[\frac{(1+\rho P^M_{t+1})}{\Pi_{t+1} P^M_t} \right]$) to match its evaluation of the economic distortions associated with the Phillips curve and government budget constraint. Distortions it may weigh differently from the fiscal authority due to inflation aversion and/or a different degree of myopia.

Therefore, the central bank, taking fiscal policy as given, sets *ex ante* real interest rates to achieve condition (24) based on its evaluation of the costs associated with the constraints they face. The resulting equilibrium real interest rate is then consistent with the tax smoothing element of the fiscal authority's foc for debt, equation (21). The central bank's attitude to inflation, therefore, impacts debt accumulation through two channels. Firstly, inflation aversion reduces the central bank's desire to generate surprise inflation to boost output and/or deflate the real value of debt. In equilibrium this reduces the rate of inflation at each potential level of debt. The fiscal authority therefore has less incentive to reduce debt to reduce inflation. Secondly, by setting real interest rates in line with their evaluation of the economic distortions contained in the economy, the central bank tilts the tax smoothing incentives of the fiscal authority, thereby impacting the pace of fiscal consolidation.

3.3 The Nash Equilibrium

The Nash solution to this simultaneous moves game is then obtained by simultaneously solving the FOCs for both policymakers alongside the three constraints. Therefore, we solve for the following thirteen variables, $\{C_t, Y_t, \Pi_t, b_t, \tau_t, G_t, P^M_t, \lambda^m_{1t}, \lambda^m_{2t}, \lambda^m_{3t}, \lambda^f_{1t}, \lambda^f_{2t}, \lambda^f_{3t}\}$ using the three constraints, the bond pricing equation (7), and the nine first-order conditions: (17)-(24), plus the central bank's FOC for output. Specifically, we need to find these thirteen time-invariant Markov-perfect equilibrium policy functions which depend on the three state variables $\{b_{t-1}, \beta_t, \epsilon_t\}$. That is, we need to find policy functions such as $b_t = b\,(b_{t-1}, \beta_t, \epsilon_t)$, $\tau_t = \tau\,(b_{t-1}, \beta_t, \epsilon_t)$, and $\Pi_t = \Pi\,(b_{t-1}, \beta_t, \epsilon_t)$ for each endogenous variable.

Intuitively, the Nash equilibrium is defined by a monetary policy that is consistent with the central bank's evaluation of the path of distortions implied by fiscal policy and a fiscal policy that sets its instruments given the bond yields (and implied debt service costs) determined by monetary policy.

4 Intuition - 2 Period Model

Before solving the complex problem defined above, in this section, we consider a far simpler model adapted from Leeper and Leith (2016), but where we consider the strategic interactions between an inflation-averse central bank and a separate fiscal authority. Similar forces will be at play in our much richer economy. The economy is a perfect foresight endowment economy with no government consumption such that consumption always equals its endowment (which is assumed constant at γ). Households can save in the form of one period bonds, such that their budget constraint in period $t = 1$, is given by,

$$Q_{1,2}b_1 = \gamma - c_1 - \tau_1 + \nu_1 b_0 \tag{25}$$

where the state variables are defined as, $b_j \equiv B_j/P_j$ reflecting the quantity of zero coupon nominal bonds issued in period j which mature one period later, deflated by the price level in period j, $\nu_t = \Pi_t^{-1}$ is the inverse of the gross rate of inflation and $Q_{t,t+1}$ is the price of zero coupon debt in period t, which matures in period $t+1$. There is an endowment, γ, in each period, which finances consumption c_1, taxation τ_1 and net savings. Therefore the household inherits a stock of one-period bonds which were issued in the previous period, b_0, and decides how much to consume, c_1, alongside the quantity of bonds to purchase in period $t = 1$, b_1.

The corresponding period $t = 2$ constraint is,

$$\tau_2 = \gamma - c_1 + \nu_2 b_1 \tag{26}$$

where it is no longer possible to issue or purchase bonds as the economy ends.

The household maximizes utility,

$$\sum_{t=1}^{2} \beta^{t-1} u(c_t) \tag{27}$$

subject to the series of budget constraints. Given the resource constraint implies consumption in each period is constant and equal to the household endowment, $c_t = \gamma$, the bond pricing equations reduce to,

$$\beta \nu_{t+1} = Q_{t,t+1} \tag{28}$$

Bond prices reflect the household's desire to earn a real return of β^{-1}, such that they are required to compensate for expected inflation of ν_{t+1}^{-1}. [15]

[15] Note that since *ex ante* real returns are tied down by preferences, the inflation-averse central bank only affects debt accumulation by affecting the inflationary bias problem associated with a given level of debt. In our more general model the central bank will also affect real interest rates and, as a result, can tilt the fiscal authority's incentives to smooth taxes.

The government's budget constraints then mirror those of the household, in period $t = 1$,

$$\beta\nu_2 b_1 = -\tau_1 + \nu_1 b_0 \tag{29}$$

and the final period $t = 2$,

$$\tau_2 = \nu_2 b_1 \tag{30}$$

Following Leeper and Leith (2016) it is assumed that inflation and taxation are costly, such that social welfare is given by,

$$-\sum_{t=1}^{2} \beta^{t-1} \left(\tau_t^2 + \theta^i (\nu_t - 1)^2\right) \tag{31}$$

where τ_t is the tax rate and $\nu_t = \Pi_t^{-1}$ is the inverse of the gross rate of inflation. The parameter θ^i captures the relative cost of inflation for the monetary (θ^M) and fiscal (θ^F) authority, respectively. This can be, more generally, thought of as the welfare cost of the inflation bias problem, which in this simple model will be associated with the desire to reduce the real value of debt rather than boosting the size of the real economy. $\theta^M > \theta^F$ implies that the central bank is independent and more inflation-averse than society/the government.

We now consider optimal policy under cooperative commitment and contrast that with the time-consistent policy with and without central bank independence.

4.1 Commitment

Assuming $\theta^M = \theta^F = \theta$ commitment policy is simple to characterize using the following Lagrangian,

$$\begin{aligned}\mathcal{L} = &\sum_{t=1}^{2} \beta^{t-1} [-\frac{1}{2} \left(\tau_t^2 + \theta(\nu_t - 1)^2\right)] \\ &+ \lambda[b_0\nu_1 - \tau_1 - \beta\tau_2]\end{aligned}$$

with FOCs for taxation of,

$$\tau_t = -\lambda \text{ for } t = 1, 2$$

and deflation,

$$-\beta\theta(\nu_2 - 1) = 0 \text{ i.e. } \nu_2 = 1$$

$$-\theta(\nu_1 - 1) = -\lambda b_0 = \tau_1 b_0$$

These imply that under commitment, pure tax smoothing is applied. Inflation is only generated to the extent that the time $t = 0$ policy maker inherits debt from the previous period, $b_0 > 0$. In which case,

$$(\frac{1}{\nu_1} - 1) = \theta^{-1} \frac{b_0^2}{(1 + \beta)}$$

and,

$$\tau_1 = \tau_2 = \frac{b_0\nu_1}{(1 + \beta)}$$

4.2 Simultaneous Moves Nash Equilibrium

By focusing on a two-period model, we can tractably analyze the time-consistent policy problem by backward induction. To do so, we solve the period $t = 2$ problem and use the resultant FOC as an incentive compatibility constraint (ICC) for the problem in period $t = 1$, which can be analyzed as a strategic game between the two policymakers subject to this additional constraint. This bypasses the need to solve for the policy functions for each endogenous variable as a function of the states, as we do for the main model.

4.2.1 Period $t = 2$ Problem

Consider the period $t = 2$ problem which maximizes social welfare in the final period, subject to the budget constraint,

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\left(\tau_2^2 + \theta^F(\nu_2 - 1)^2\right) \\ &+ \lambda_2\left[-b_1\nu_2 + \tau_2\right]\end{aligned}$$

with FOCs for taxation,

$$-\tau_2 + \lambda_2 = 0$$

and deflation,

$$-\theta^F(\nu_2 - 1) - b_1\lambda_2 = 0$$

Combining these FOCs we obtain,

$$\theta^F\nu_2(1 - \nu_2) = \tau_2^2 \tag{32}$$

This can be combined with the budget constraint to obtain the solution for taxation and deflation,

$$\tau_2 = \frac{\theta^F b_1}{\theta^F + b_1^2} \text{ and } (1 - \nu_2) = \frac{b_1^2}{\theta^F + b_1^2}$$

which describes the balance between inflation and taxation in period 2, given the need to pay off the debt inherited from period 1. It should be noted there are no strategic interactions here. The central bank sets the short-run interest rate between periods 1 and 2, but in period 2, all that remains is to pay off the remaining debt stock. This debt stock will imply a combination of taxation and inflation in period 2; however, period 2 inflation does nothing to reduce the debt burden since it was fully anticipated when the period 1 debt was issued. For this reason, under commitment, the policymaker would commit to not generating any inflation in period 2. However, in the absence of commitment, there is a debt-driven inflationary bias problem. Period 2 inflation expectations rise until the period 2 policymaker does not wish to generate any further inflation surprises beyond this. In period 1 there is then a debt stabilization problem in that reducing debt today will help offset the tax and inflation costs tomorrow. We will now look at how central bank independence affects this debt stabilization bias problem.

4.2.2 Period $t = 1$ Problem

The policymakers implementing policy in period $t = 1$ will consider that the period $t = 2$ government will behave in the way just described. The intertemporal budget constraint facing the period $t = 1$ policy maker is given by,

$$b_0 \nu_1 = \tau_1 + \beta \tau_2$$

Monetary Authority:

The period t=1 problem for the monetary authority is given by,

$$\begin{aligned}\mathcal{L} = &\sum_{t=1}^{2} (\beta^M)^{t-1} [-\frac{1}{2}\left(\tau_t^2 + \theta^M (\nu_t - 1)^2\right)] \\ &+ \mu^M \left[b_0 \nu_1 - \tau_1 - \beta \tau_2\right] \\ &+ \beta^M \lambda^M \left[-\theta^F \nu_2 (\nu_2 - 1) - \tau_2^2\right]\end{aligned}$$

which differs from the problem under commitment in three ways (1) the monetary authority may be more inflation averse, $\theta^M \geq \theta^F$, (2)they take the fiscal authority's tax policy in period 1, τ_1 as given and (3)they cannot make commitments about future behavior and must respect the period 2 ICC, equation (32).

The associated FOCs are as follows, firstly for taxation in period $t = 2$, τ_2,

$$-\tau_2 - \frac{\beta}{\beta^M} \mu^M - 2\tau_2 \lambda^M = 0$$

deflation in period 1,

$$-\theta^M \nu_1 (\nu_1 - 1) + \mu^M (\tau_1 + \beta \tau_2) = 0$$

and period 2,

$$-\theta^M (\nu_2 - 1) - \lambda^M \theta^F (2\nu_2 - 1) = 0$$

The FOCs can be rearranged as,

$$\lambda^M = \frac{\theta^M}{\theta^F} \frac{(1 - \nu_2)}{(2\nu_2 - 1)} \tag{33}$$

$$\mu^M = -\tau_2 (1 + 2\lambda^M) \frac{\beta^M}{\beta} \tag{34}$$

and,

$$(1 - \nu_1) = -(\theta^M)^{-1} \mu^M b_0 \tag{35}$$

Using the budget constraint, (29), and the ICC, (32) we can eliminate next period's taxation and deflation from (33)-(35) and after solving, we obtain the monetary authority's strategy, $\nu_1 = f^M(\tau_1, b_0)$ conditional on the Fiscal authority's chosen value of τ_1 and the inherited level of debt, b_0. This defines the central bank's reaction function.

Fiscal Authority:

Considering the problem of the fiscal authority we have,

$$\begin{aligned}\mathcal{L} = &\sum_{t=1}^{2}(\beta^F)^{t-1}[-\frac{1}{2}\left(\tau_t^2+\theta^F(\nu_t-1)^2\right)]\\ &+\mu^F\left[b_0\nu_1-\tau_1-\beta\tau_2\right]\\ &+\beta^F\lambda^F\left[-\theta^F\nu_2(\nu_2-1)-\tau_2^2\right]\end{aligned}$$

Here, the fiscal authority takes the central bank's monetary policy (the value of ν_1) as given when setting its tax policy in period 1, and is also constrained by the ICC in period 2. The associated FOCs are as follows, firstly for taxation in period $t=1$,

$$-\tau_1-\mu^F=0$$

and period t=2,

$$-\tau_2-\frac{\beta}{\beta^F}\mu^F-2\tau_2\lambda^F=0$$

and deflation in period t=2,

$$-\theta^F(\nu_2-1)-\lambda^F\theta^F(2\nu_2-1)=0$$

These can be rearranged as,

$$\lambda^F=\frac{(1-\nu_2)}{(2\nu_2-1)} \tag{36}$$

$$\mu^F=-\tau_2(1+2\lambda^F)\frac{\beta^F}{\beta} \tag{37}$$

and

$$\begin{aligned}\tau_1 &= -\mu^F \\ &= \tau_2(1+2\lambda^F)\frac{\beta^F}{\beta}\end{aligned} \tag{38}$$

Therefore, the Fiscal Authority deviates from tax smoothing by raising taxation in period 1 relative to period 2, depending on the size of λ^F. Again, using the budget constraint, (29), and the ICC, (32) we can eliminate next period's taxation and deflation from the FOCs (36)-(38). Solving these equations gives us the solution for $\tau_1 = f^F(\nu_1, b_0)$ i.e. the fiscal authority's chosen tax rate as a function of the initial debt stock and the inflation chosen by the monetary authority. This constitutes the fiscal authority's reaction function.

Figure 1 plots these reaction functions and the associated Nash equilibrium for the case where $\theta^M=\theta^F=1$ (the solid lines) and for $\theta^M=3>\theta^F=1$ (the dashed line). The blue lines are the fiscal authority's policies given the initial rate of inflation generated by central bank monetary policy. The magenta lines are the monetary authority's policies conditional on the fiscal authority's first-period tax rate. Where these reaction functions cross in the first column defines the Nash equilibrium. It is the same in both plots but with the axes reversed. From the first column, we can see that the lower the first-period tax rate levied by the fiscal authority, the greater the inflation generated by the central bank. The same is true of the fiscal authority - the greater the inflation of the central bank, the

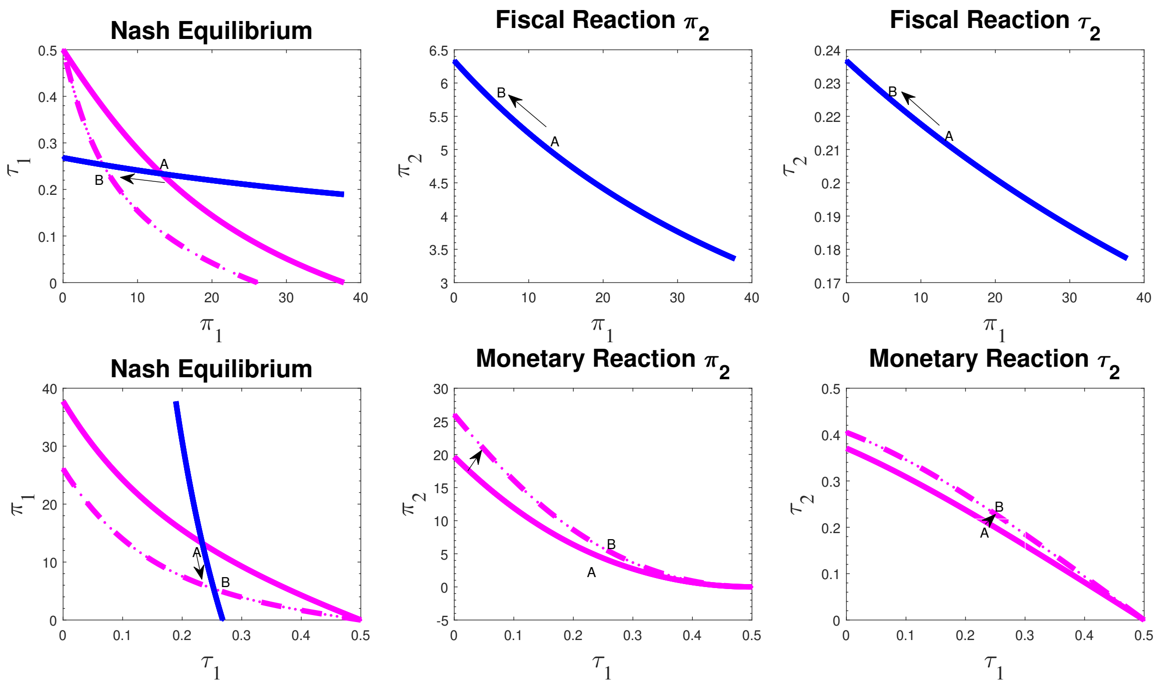

Figure 1: Nash Equilibrium Under Central Bank Independence

lower the taxation levied by the government. Making the central bank more inflation-averse shifts their reaction function, implying they reduce inflation in the first period, given the tax rate chosen by the government. As a result, the simultaneous moves Nash equilibrium implies a significantly lower rate of inflation in the first period, and a modest increase in the tax rate. Thus, the extent to which the debt is reduced in the first period falls - inflation has fallen, and the debt stabilization bias has been softened. However, the higher debt left in the second period raises taxation and inflation in period 2. So, making the central bank independent and inflation-averse improves outcomes in period 1 by significantly reducing inflation but only modestly increasing taxation. However, the greater stock of debt left to period 2 worsens outcomes in period 2. Thus, there is a short-run gain to central bank independence, but eventually, by weakening the debt stabilization bias, debt levels will increase, which can lead to worse inflation outcomes in the future.

We can trace out the Nash equilibrium as we move from a cooperative but time-consistent equilibrium to increasing levels of central bank inflation aversion - see Figure 2. The solid lines are the commitment outcome, plotted for comparison. Under commitment, there is a modest level of inflation in period 1, which deflates the real value of debt. This is followed by a commitment to avoid inflation in period 2 and to equalize taxes in periods 1 and 2 to meet the intertemporal budget constraint, reflecting a policy of pure tax smoothing. In contrast, time-consistent policymakers wish to reduce debt in period 1 more aggressively than they would under commitment. Why? The more debt left to period 2, the worse the inflation bias in period 2. This desire to reduce debt quicker than found under commitment is the debt stabilization bias. As central bank inflation aversion increases first period inflation falls and first period taxation rises. Future debt levels will rise, which will raise taxation and inflation in period 2. Therefore, central bank independence can reduce inflation in the short run by mitigating the debt stabilization problem. However, as a result, debt levels are higher in period 2, worsening inflation and taxation outcomes. This dynamic is a feature of our richer model - upon granting central

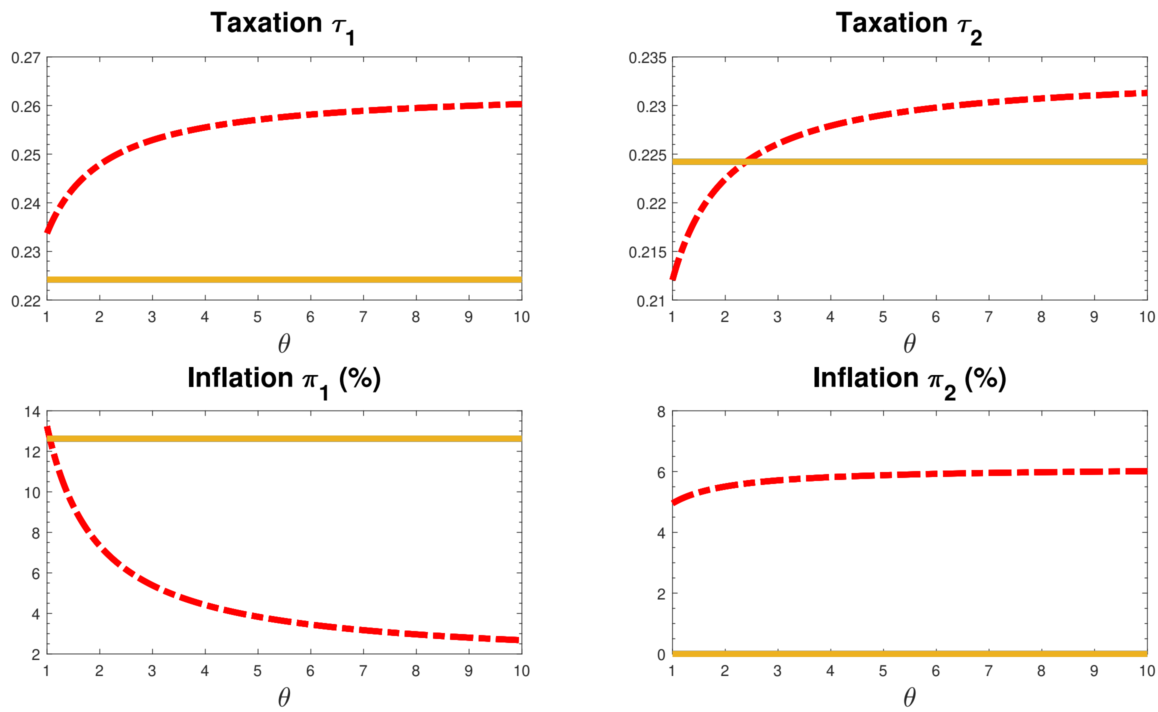

Figure 2: Impact of Inflation Aversion on Nash Equilibrium

bank independence, inflation falls initially, but debt starts to accumulate. Eventually, higher debt levels will generate worsening inflation outcomes.

We will now look at how central bank independence affects debt stabilization bias problem in our full model.

5 Numerical Analysis

5.1 Solution Method and Calibration

For the full policy problem described in section 3, the equilibrium policy functions cannot be computed in closed form. We thus resort to computational methods and derive numerical approximations to the policy rules. Local approximation methods are not applicable for this purpose because the model's steady state around which local dynamics should be approximated is endogenously determined as part of the model solution and thus *a priori* unknown. In light of this difficulty, we resort to a global solution method. Specifically, we use the method of Chebyshev collocation with time iteration to solve the model.[16] The detailed algorithm is presented in section B.2 in the appendix. In general, optimal discretionary policy problems can be characterized as a dynamic game between the private sector and successive policymakers. Multiplicity of equilibria is a common problem in dynamic games. Since we use polynomial approximations and our focus is on equilibria with continuous strategies (see Judd (2004) for a discussion) we are searching for continuous Markov-perfect equilibria where agents condition their strategies only on payoff-relevant state variables.

Before solving the model numerically, the benchmark values of structural parameters must be specified. The calibration of parameters is summarized in Table 1. We set

[16]See Judd (1998) for a textbook treatment.

$\beta = (1/1.02)^{1/4} = 0.995$, which is a standard value for models with quarterly data and implies 2% annual real interest rate. Our policymakers are relatively myopic, and in the benchmark calibration, we assume $\beta^F = \beta^M = 0.982$, which implies that both policymakers consider a 20-year time horizon relative to household preferences. We shall consider the implications of varying the myopia of both policymakers, separately and together, in section 5.3, below. The intertemporal elasticity of substitution is set to one-half ($\sigma = \sigma^g = 2$), which is in the middle of the parameter range typically considered in the literature. Labor supply elasticity is set to $\varphi^{-1} = 1/3$. The elasticity of substitution between intermediate goods is chosen as $\epsilon_t = 14.33$, which implies a monopolistic markup of approximately 7.5%, similar to Siu (2004). The decay parameter defining the geometric maturity structure of government debt, $\rho = 0.95$, corresponds to $4 \sim 5$ years of debt maturity, consistent with US data. The scaling parameter $\chi = 0.007$ ensures that the share of government spending in output is about 7.8%. The cost push shock parameters are set to $\rho_a = 0.95$ and $\sigma_a = 0.01$. The price adjustment cost parameter $\phi = 50$ - implying that on average, firms re-optimize prices every 6 months - is in line with empirical evidence. Transfers are fixed at a level that amounts to 9% of steady-state GDP in the benchmark calibration.

With this benchmark parameterization, we solve the fully nonlinear models via the Chebyshev collocation method. The maximum Euler equation error over the full range of the grid is of the order of 10^{-6}. As suggested by Judd (1998), this order of accuracy is reasonable.

5.2 Numerical Results

In this section, we explore the properties of the equilibrium under the optimal time-consistent policy. We begin by considering the steady state under our benchmark calibration without any strategic interactions between the central bank and the fiscal authorities. This corresponds to the case considered in Leeper et al. (2021), which also considers the impact of debt maturity, price stickiness, and markups on that steady state. Here, instead, we are interested in how providing the central bank with an anti-inflation mandate, which they can pursue with instrument independence, impacts equilibrium outcomes.

In our first experiment, we gradually increase the degree of inflation aversion, α_π, from 0 to 0.95 for the central bank only.[17] All other parameters remain the same as in the benchmark calibration. In the first subplot of Figure 3, we plot two values for inflation. The first is the steady-state value of inflation implied by the given value of α_π, the second is the initial value of inflation that would occur where we unexpectedly grant the central bank independence, having previously been in the initial steady state where there was no central bank independence ($\alpha_\pi = 0$). The second subplot reveals the steady-state level of debt that would emerge for the various degrees of central bank inflation aversion. The results are fully consistent with those obtained for the simple two-period model analyzed above. Upon appointing a conservative central bank for a given level of government debt, the inflation bias problem is reduced, and inflation falls. However, since there is a lower inflationary cost associated with government debt, the fiscal authorities are less inclined to act to reduce debt and allow debt to rise above the level they would have supported without central bank independence. This means that the steady-state level of debt rises with the level of central bank inflation aversion, and steady-state inflation does, too.

[17] Moving much beyond $\alpha_\pi = 0.95$ reduces numerical accuracy and as we approach strict inflation targeting, $\alpha_\pi = 1$, debt dynamics appear to be unstable.

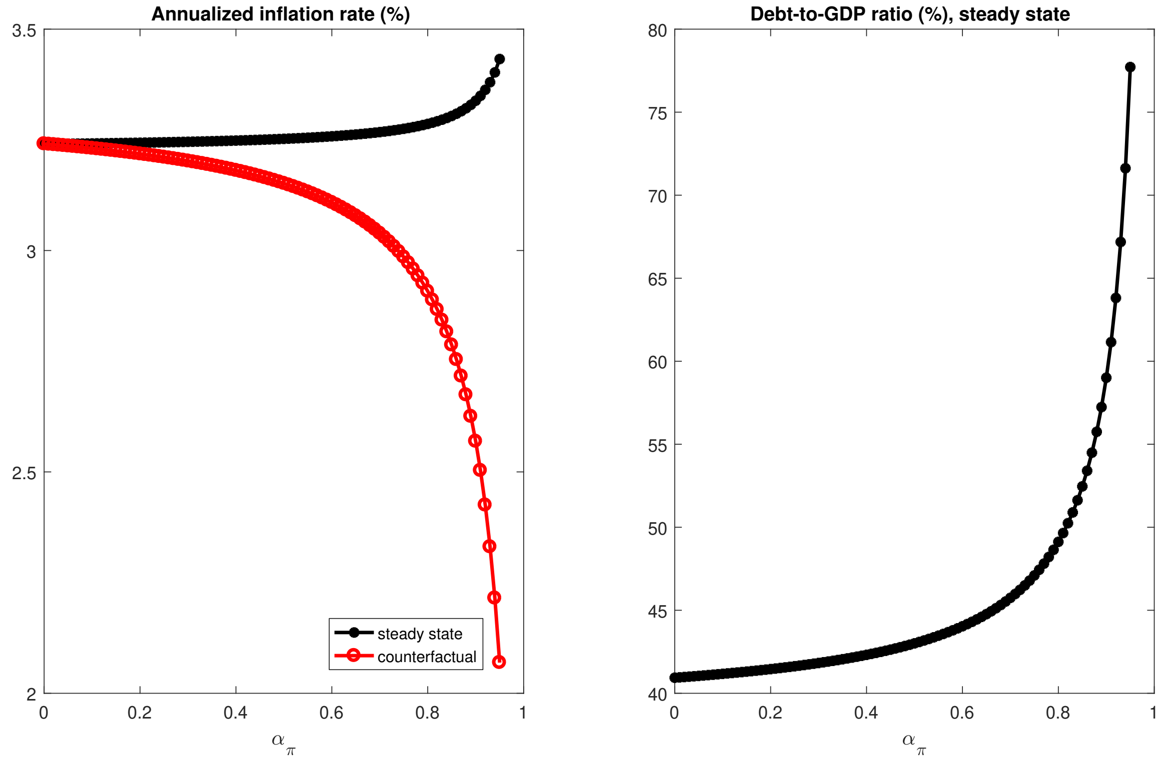

Figure 3: Increasing Inflation Aversion of Independent Central Bank

We can illustrate what happens dynamically upon granting central bank independence in the simulation plotted in Figure 4. Here, the degree of central bank inflation aversion is $\alpha_\pi = 0.95$. Upon granting independence, there is an immediate fall in inflation, as the inflation bias is reduced thanks to the independent central bank's strong degree of inflation aversion. This allows the central bank to relax monetary policy with a fall in real interest rates. Lower inflation also reduces the welfare costs of debt for fiscal authorities, and lower real interest rates tilt the tax smoothing part of optimal debt policy towards delayed consolidation. As a result the government can now relax fiscal policy, cutting taxation substantially and raising public consumption. This gradually increases government debt. Over time, this worsens the inflation bias problem until eventually inflation is higher than it was before independence, and the fiscal authority has to increase taxation to support the higher debt level. It should be noted that this process is gradual - given the quarterly time interval of the model, it takes over two decades for inflation outcomes to worsen relative to their pre-independence level.

We can also examine the welfare implications of central bank independence by plotting the policy functions for a measure of social welfare with and without central bank independence - see Figure 5. Welfare is measured as the discounted value of household utility conditional on the economy inheriting particular combinations of the state variables, debt, b_{t-1} and the autocorrelated elasticity of substitution, ϵ_t. Therefore, it measures the utility the household would experience as the economy transitions from that point in the state space to the steady state. Again, the degree of central bank independence considered is $\alpha_\pi = 0.95$. For all values of the state space considered, both in terms of mark-up shocks and levels of debt, the welfare under central bank independence is higher. That this would be the case in the early days of independence when inflation is lower than it would have been without independence is intuitive. While independence leads to increased debt and inflation in the long run, its welfare implications are less clear. How-

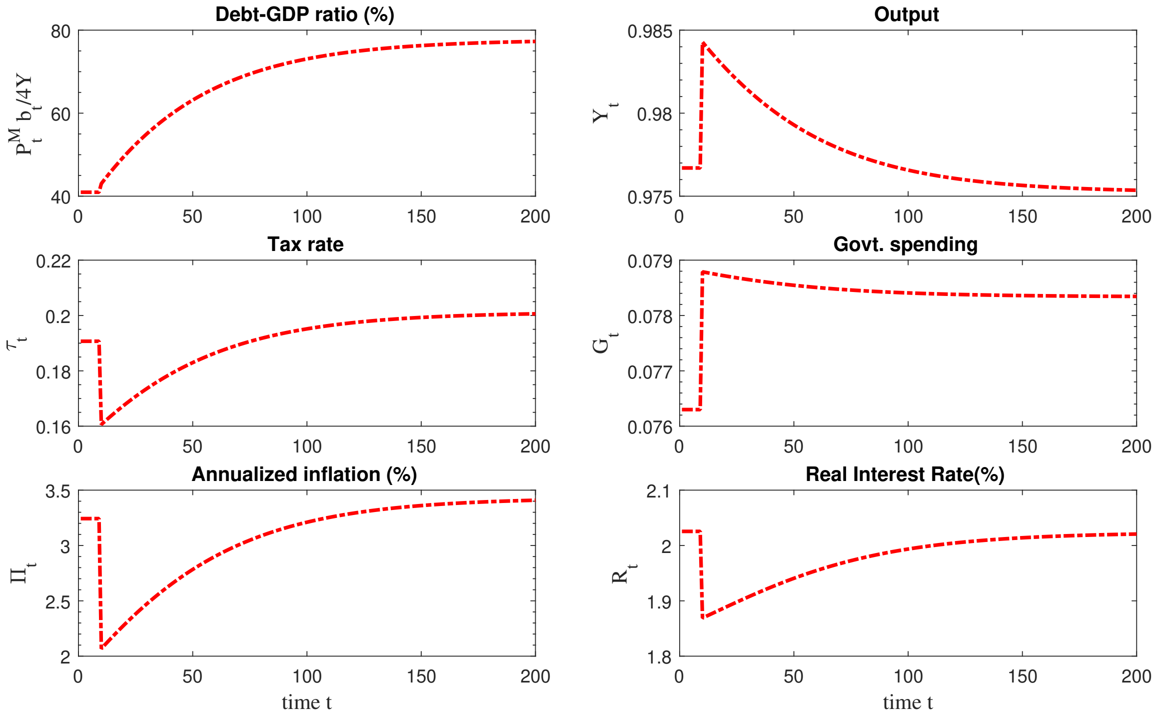

Figure 4: Impact of Granting Central Bank Independence

ever, losing independence after debt levels rise would result in even higher inflation and a need for aggressive fiscal consolidation to restore debt to its pre-independence levels. This would be very costly in terms of welfare. Therefore, although central bank independence does not give the bank free reign to control inflation, it still offers an improvement over time-consistent but coordinated policy for all points in the state space.

We can compare the policy of central bank independence as a form of inflation control with what would happen if, instead of appointing a conservative central bank, both policymakers adopted a common anti-inflation stance and acted in a coordinated manner. To do so, Figure 6 replicates Figure 3, but with a common degree of inflation aversion for both policymakers. Here, the figure is very similar to that where only the central bank is inflation averse, although it is possible to see a larger accumulation of debt as inflation aversion rises. In this sense, the increase in debt upon central bank independence would be worse if the government shared the central bank's inflation aversion. This is then reflected in the welfare costs of inflation aversion - a coordinated increase in inflation aversion is actually costly in welfare terms - see Figure 7 which shows that for any combination of debt and shock, social welfare is greater without the government and central bank adopting the same aversion to inflation and coordinating policy.

Therefore, central bank independence is welfare improving despite the long-run costs associated with the higher levels of debt and inflation it creates. However, encouraging the government to share the central bank's inflation aversion actually worsens outcomes.

5.3 Myopia

We now turn to consider the impact of policymaker myopia on equilibrium outcomes. We first examine the case without any inflation aversion on the part of either policy maker, $\alpha_\pi = 0$, but allow for three possible permutations of β^M and β^F: (1) $\beta^M = \beta^F = \beta = 0.989$, (2) $\beta^M = 0.982 < \beta^F = 0.989$ and (3) $\beta^F = 0.982 < \beta^M = 0.989$.

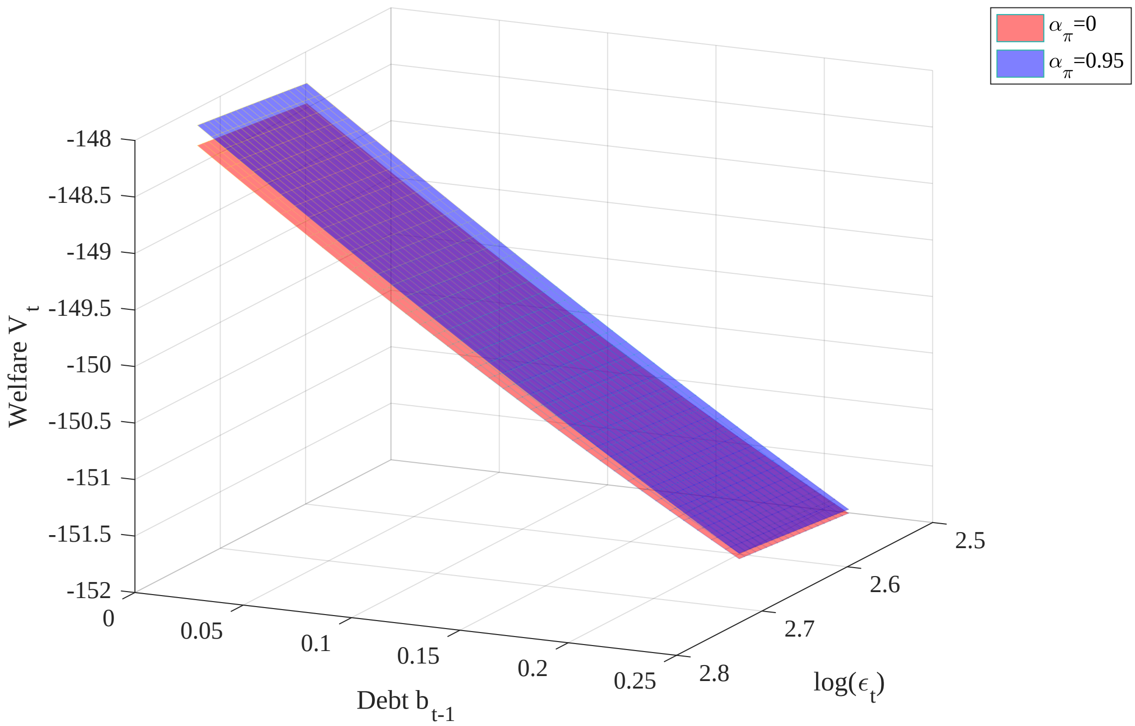

Figure 5: Welfare Gains from Central Bank Independence Across State Space

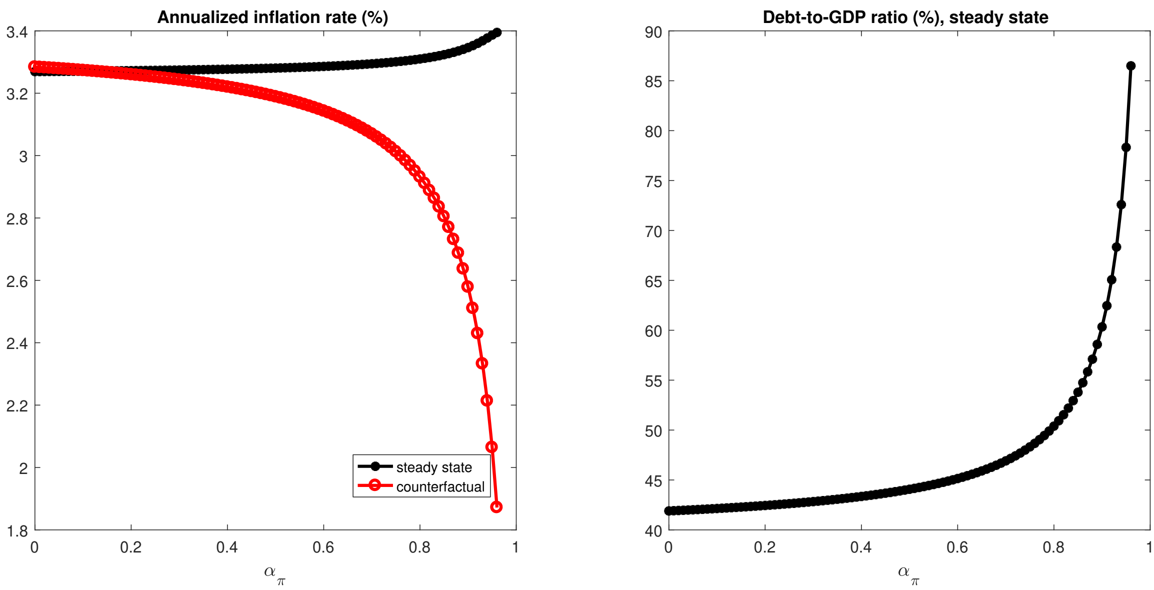

Figure 6: Coordinated Inflation Aversion

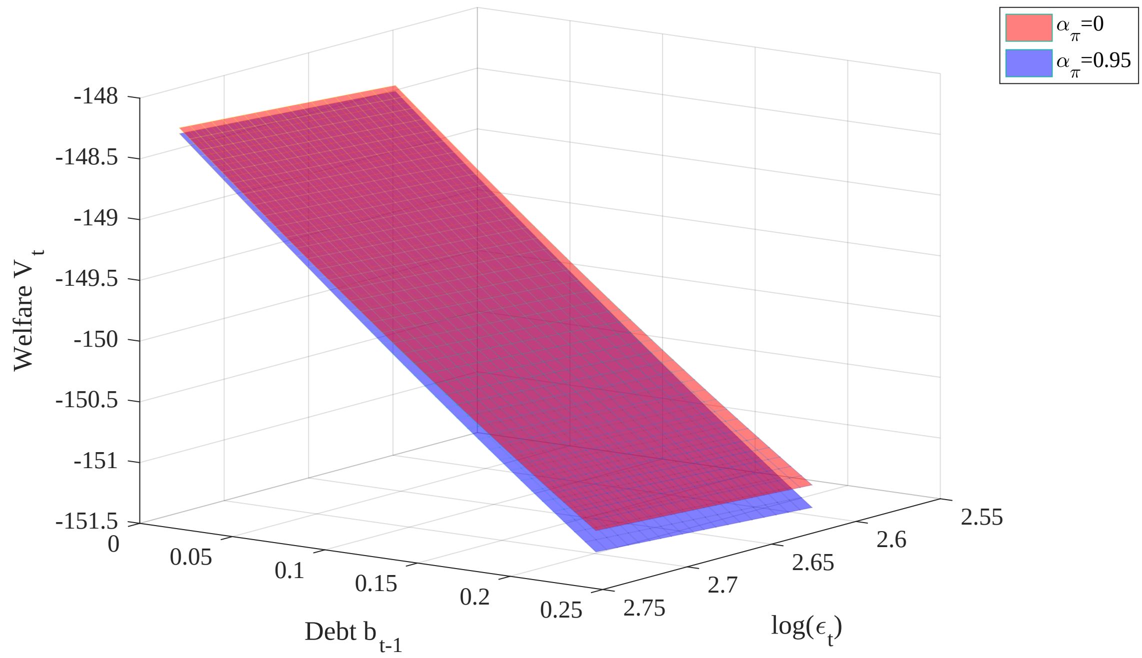

Figure 7: Coordinated Increase in Inflation Aversion

In other words, we are reducing myopia in one policymaker at a time and then both simultaneously. We then assume we were in the initial steady-state where $\beta^M = \beta^F = 0.982$ before, unexpectedly, one of these three cases emerges. We, therefore, capture the dynamic path from the old steady-state with a high degree of myopia to the new, with a lower degree of myopia, in each case - see Figure 8. We find that, as in Leeper et al. (2021) in the context of a single policymaker, decreasing myopia across both policymakers simultaneously decreases steady-state debt and inflation in the long run. In this case, the steady-state level of debt actually turns negative, and the government accumulates assets. However, there is a short period of increased inflation during the transition to the lower steady-state debt level, as the coordinating policymakers wish to reduce the level of debt they inherited since the marginal long-run inflationary costs of that debt now weigh more heavily in the policy maker's calculus and this fuels the inflation bias associated with that debt. Interestingly, making the fiscal authority less myopic than the monetary authority actually increases the steady-state debt but reduces inflation. The monetary authority sets interest rates in line with its intertemporal evaluation of the costs of government debt, implying that it feels free to reduce rates when the fiscal authority is less myopic, leading to a tilting in the tax smoothing element of fiscal policy towards higher debt. In contrast, reducing myopia in monetary policy leads to a very large increase in inflation until government debt has been reduced. Intuitively, a relatively myopic monetary authority cares less about future distortions relative to current distortions, allowing policy to counteract current inflation aggressively without worrying that this will give rise to a rising path for debt which will ultimately undo their anti-inflation stance. When they anticipate that such a policy will raise inflation in the long run a less myopic central bank moderates monetary policy today - the increase in real interest rates relative to inflation during the transition to a low debt economy is much less when the monetary authority is less myopic, especially relative to the fiscal authority.

In Figure 9, we re-do this experiment but assume the central bank (alone) has a degree

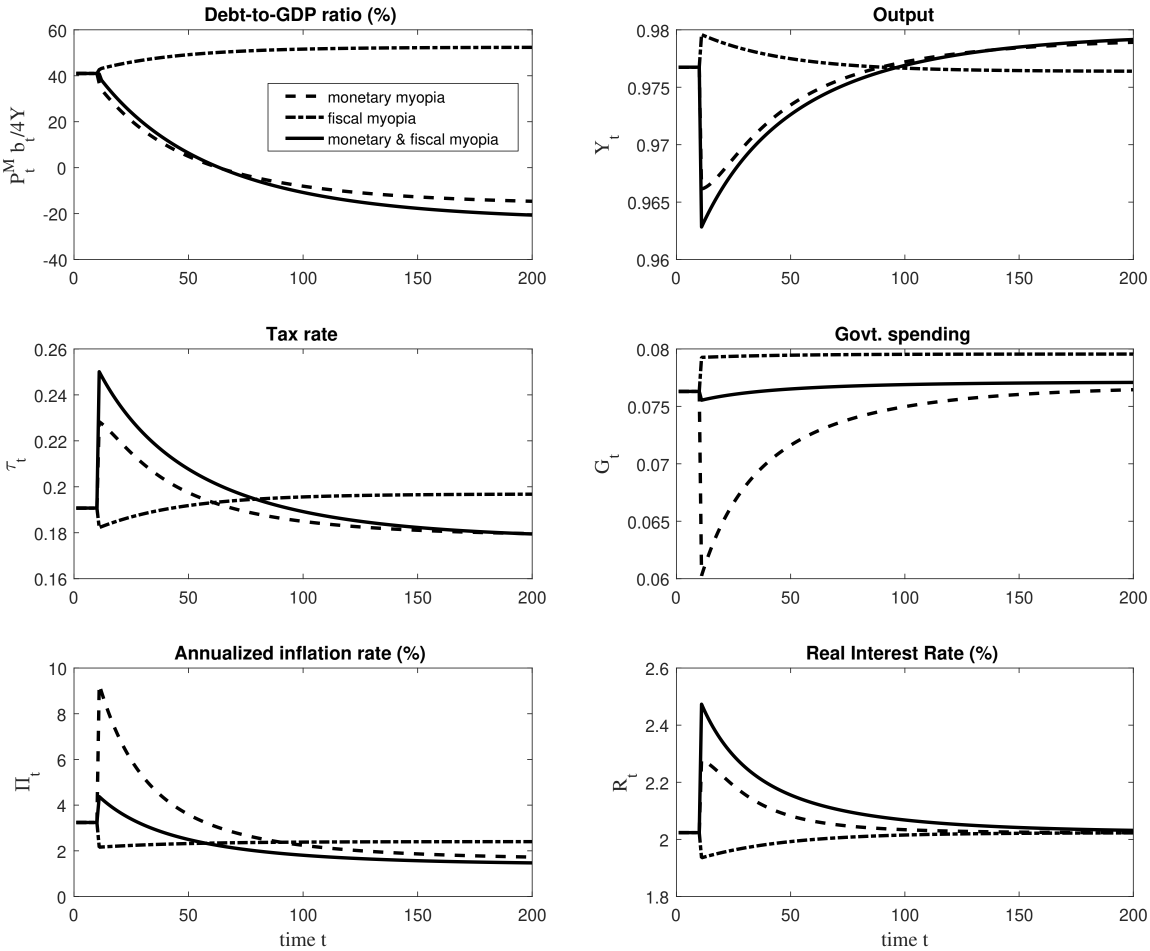

Figure 8: Decreasing Policy Maker Myopia

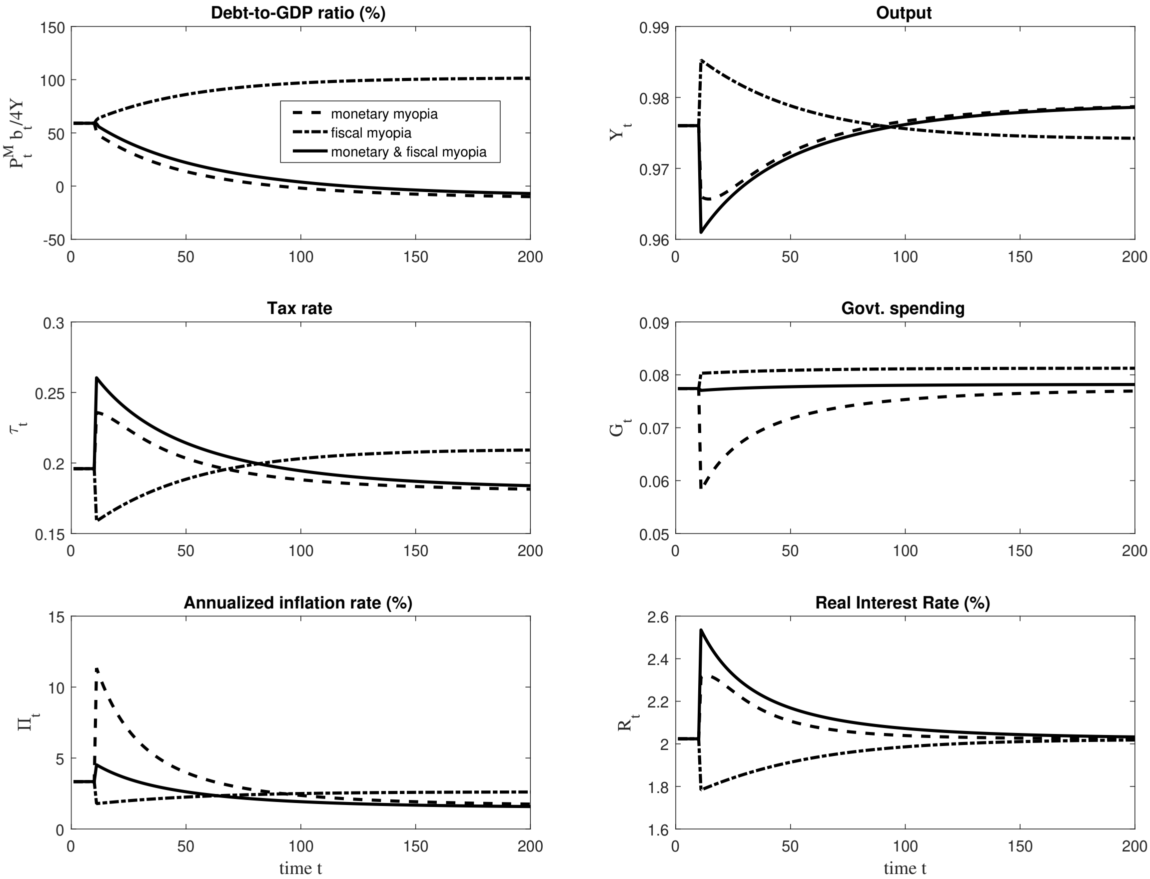

Figure 9: Decreasing Policy Maker Myopia with an Inflation Averse Central Bank

of inflation aversion, $\alpha_\pi = 0.9$. The impact of changing myopia is similar to that observed before - decreasing the myopia of the monetary authority, either alone or in conjunction with the fiscal authority leads to both the greatest reduction in steady-state debt and the the greatest increase in inflation during the transition to that steady state. While much of the literature emphasizes political frictions in fiscal policymaking—often prioritizing short-term concerns over the long-term consequences of current policies—this friction has only modest effects on debt and inflation.

Given that the conservatism of the central bank encourages the fiscal authority to issue debt, which undermines the central bank's fight against inflation, we now turn to assess whether or not debt targets can return the control of inflation to the central bank.

5.4 Debt Targeting

Our analysis suggests that even an operationally independent conservative central bank does not have complete control over inflation, particularly in the longer term. Increasing conservatism does reduce inflation for a sustained period, but eventually, reduced fiscal discipline will undermine the central bank's stance, even though the central bank never abandons its conservatism and never attempts to bail out the government. It is possible that other devices could be used to help the central bank regain control over inflation. For example, Eggertsson (2013) and de Beauffort (2024), assume that the bank ignores the government's budget constraint in formulating their policy. This is a stronger assumption than assuming that the central bank does not succumb to a regime of Fiscal Dominance.

Instead, the central bank implicitly commits to ignoring the fiscal repercussions of their policy, even though government debt is an endogenous state variable that conditions economic agents' inflation expectations. An alternative approach to facilitate the separation of monetary and fiscal policies might be for the fiscal authority to impose additional fiscal discipline on itself in the form of a debt target, in a mirror of the central bank's inflation target. In this case, the fiscal authority's flow objective function becomes,

$$U_t^F = (1-\alpha_b)\left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\chi G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi}\right) - \frac{\alpha_b}{2}\left(\frac{P_t^m b_t}{Y_t}\right)^2 \tag{39}$$

Therefore, just as increasing α_π measures the central bank's aversion to inflation beyond that implied by social welfare alone, α_b does the same for the government's aversion to debt. Can this enhanced fiscal discipline liberate the central bank in pursuing its inflation objective?

To assess this, we re-derive the fiscal authority's FOCs in the presence of a debt aversion term in this form in Appendix (B.3). We then re-solve the benchmark model with a range of values of debt aversion (α_b). We find that a modest degree of debt aversion, $\alpha_b = 0.0002$ maximizes the expected social welfare gains from joint reform of central bank independence combined with the fiscal authority adopting an aversion to debt accumulation across a wide range of central conservatism. Fig. 10 then replicates Fig. 3 by plotting both the initial and steady-state value of annualized inflation against the central bank's inflation aversion parameter, α_π. The second subplot plots the steady-state debt-to-GDP ratio against the same parameter. Starting from a point with no inflation aversion, $\alpha_\pi = 0$, we can isolate the impact of the fiscal authorities becoming debt averse. Inflation falls on impact from 3.24% to 2.72%, and then gradually falls further to 2.49%, as debt falls from 41% to 35% of GDP. The aversion to debt reduces debt, which mitigates the inflation bias problem in the long run. Through expectations, this reduces current inflation, although it remains above its new steady-state value until the fiscal authorities have succeeded in their desire to reduce debt. The tilting of inflation towards the present as debt falls under debt aversion is the opposite of what happens under inflation conservatism, where inflation rises over time as inflation conservatism discourages fiscal discipline. This suggests that combining the two may balance these two effects, which is exactly what we see. Keeping debt aversion fixed at $\alpha_b = 0.0002$, increasing central bank conservatism consistently reduces inflation in both the short and long run. The particular value for debt aversion which maximises social welfare $\alpha_b = 0.0002$ achieves this by balancing the the desire to reduce debt due to the aversion to debt against the desire to increase debt due to the lower inflation generated by central bank conservatism. As a result, the path for inflation is lower while the debt remains close to its original level as we increase central bank conservatism alongside this particular degree of debt aversion.

6 An Unexpected Crisis

Our benchmark calibration (and any of the variants considered above) imply very limited movements in government debt in the face of our mark-up shock - The standard deviation of the debt-to-GDP ratio in our benchmark calibration is only 0.3%. The same is true for technology shocks. However, shifts in myopia and/or the degree of central bank

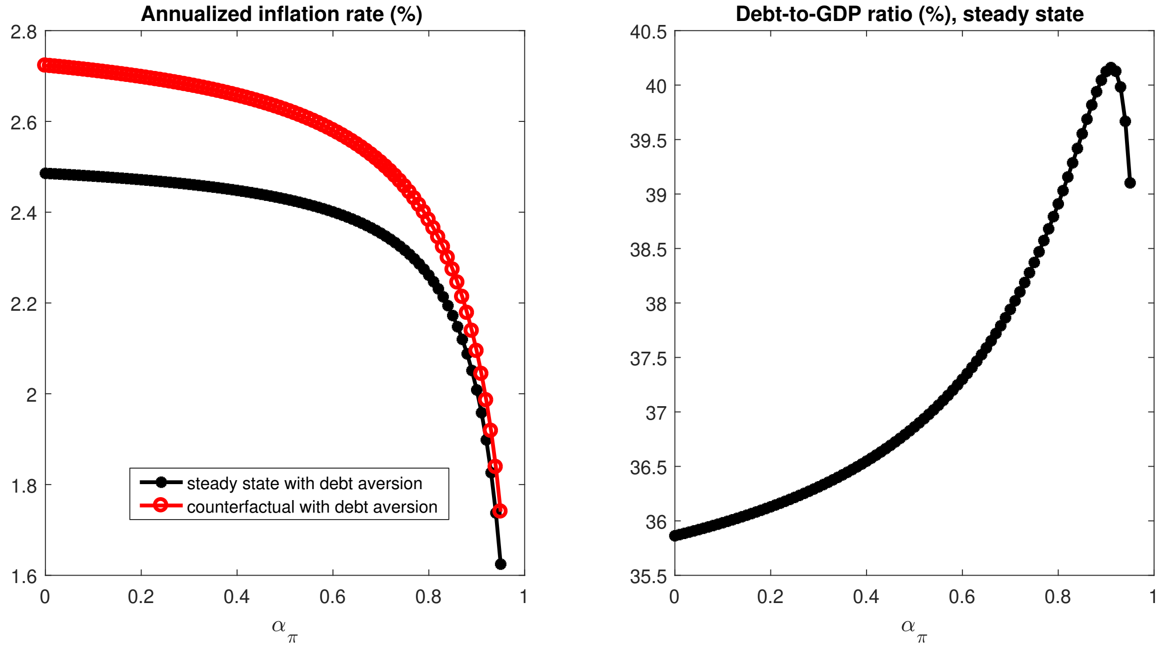

Figure 10: Increasing central bank independence, with debt aversion

independence could explain larger movements in debt. It is not obvious, however, that central banks have become more inflation-averse in recent years to the extent necessary to explain the large increases in debt we have observed.[18] However, there have been some unusually large shocks, which may have driven the observed increase in government debt: significant falls in GDP during the financial crisis and pandemic; large increases in government transfers at the same time; and large falls in government debt yields in line with a flight to safety during periods of crisis. To explore the impact of such effects, we split our model economy into two regimes. The first is as described in our benchmark calibration above. The second includes a variety of additional shocks that can occur either individually or together whenever the economy enters a 'crisis' episode: (1) A gradual 6% fall in productivity; (2) a 50% increase in government transfers; and (3) a flight to safety consistent with a fall in real interest rates from 2% to -2%. These are broadly in line with our experience during the financial crisis/pandemic.

Figure 11 plots the impact of these shocks when they occur individually. We assume that we are initially in the steady state consistent with a central bank with inflation aversion of $\alpha_\pi = 0.9$, before a crisis unexpectedly occurs. Prior to the first occurrence of the crisis, no one expected the crisis to occur, but afterwards, it is expected to follow a two-state Markov process, $\begin{bmatrix} p_N & 1-p_N \\ 1-p_C & p_C \end{bmatrix}$ where p_j is the probability of remaining in regime j ($j = C, N$) given we are currently regime j and $1 - p_j$ is the probability of exit to the other regime $k, j = (C, N), j \neq k$. To capture a large fall in output, we introduce time-varying productivity, A_t, such that the firm's production function becomes,

$$Y_t(j) = A_t N_t(j) \tag{40}$$

In a crisis (C), we assume that productivity falls by 2% immediately and then gradually

[18]Empirical estimates of the inflation aversion of the US Fed, for example, suggest that the Fed did not decisively turn conservative following the Volcker disinflation (see Chen et al. (2017)), and following the financial crisis has been 'less conservative' than the historical average (see Kirsanova et al. (2023)).

by a further 4% at rate $\rho^C = 0.85$. After returning to the normal (N) regime, productivity rises to its original value of 1 at a rate, $\rho^N = 0.95$. In the case of transfers, they simply jump by 50% in the crisis and return to normal immediately upon exiting the crisis. Finally, $\beta > \beta^C$ is the 'flight-to-safety' regime where households have a greater desire to save in the form of government bonds, ceteris paribus. During the flight to safety, the discount factor rises to $\beta^C = 1.005$, and in normal times, it stays at the benchmark calibration of $\beta^N = 0.995$. This is captured by adopting the following process for the private sector discount factor,

$$\beta_t = \rho^i \beta_{t-1} + \left(1 - \rho^i\right) \beta^i$$

with $i = C, N$ respectively denoting the crisis and normal regimes, $\beta^C > \beta^N$, respectively. The persistence of beta within each regime is $\rho^N = 0.95$ and $\rho^C = 0.85$. The fact that we have adopted the same values of ρ^i for both productivity and time preference shocks means that we need only track one additional state variable which captures where in the evolution of crises we are. The probabilities of each regime are given by, $p_C = 0.9859$ and $p_N = 0.9868$.

Consider the green line in Figure 11 - this is the case where the crisis implies a sharp but temporary increase in government transfers. Under a conventional tax smoothing policy, this is the classic experiment where the policy would call for a permanent rise in government debt to facilitate a smaller but sustained rise in taxation to pay for the temporary increase in transfers. Instead, what happens here is that the government raises taxes by even more than the rise in transfers so that the debt-to-GDP ratio actually falls throughout the crisis. This is despite the fact that higher taxation reduces output, the denominator of the debt-to-GDP measure. The higher taxation makes the economy less efficient, which worsens the inflation bias and fuels inflation, driving the desire to decrease government debt. Similarly, the red line considers the case of a very large fall in productivity/output. Despite a significant contraction of the tax base, a decline in government consumption and higher taxes result in a modest decrease in government debt. The output contraction is consistent with a fall in the real interest rate as households attempt to save to smooth the ongoing fall in output/consumption, but it re-normalizes relatively quickly, well before the crisis is over. Finally, the blue line considers a flight to safety, which drives real interest rates down to around -2% for the duration of the crisis, and they then recover upon exiting the crisis. This is now consistent with a large increase in government debt. There is an immediate fall in inflation as the debt stabilization bias is weaker when the policymakers are relatively myopic. The fiscal authority does not feel as constrained by the inflation consequences of its actions, such as cutting taxes and boosting public consumption. As debt accumulates towards 100% of GDP we currently observe in the US, inflation rises, reflecting the worsening of the inflation bias problem as debt increases, surpassing the pre-crisis rate of inflation after around 10 years. However, it is only when the re-normalization of interest rates begins that the consequences of this higher debt are felt fully. Inflation immediately jumps up as we return to the normal regime of positive real interest rates. Therefore, upon exiting the flight-to-safety, there is a significant tightening of monetary policy and a severe fiscal consolidation as both policymakers seek to reduce debt levels and offset the inflation generated by high levels of debt.

It is interesting to note that the risk of further crises affects the new steady state that the economy enters outside of the crisis regime. In the case of a risk of a fall in output

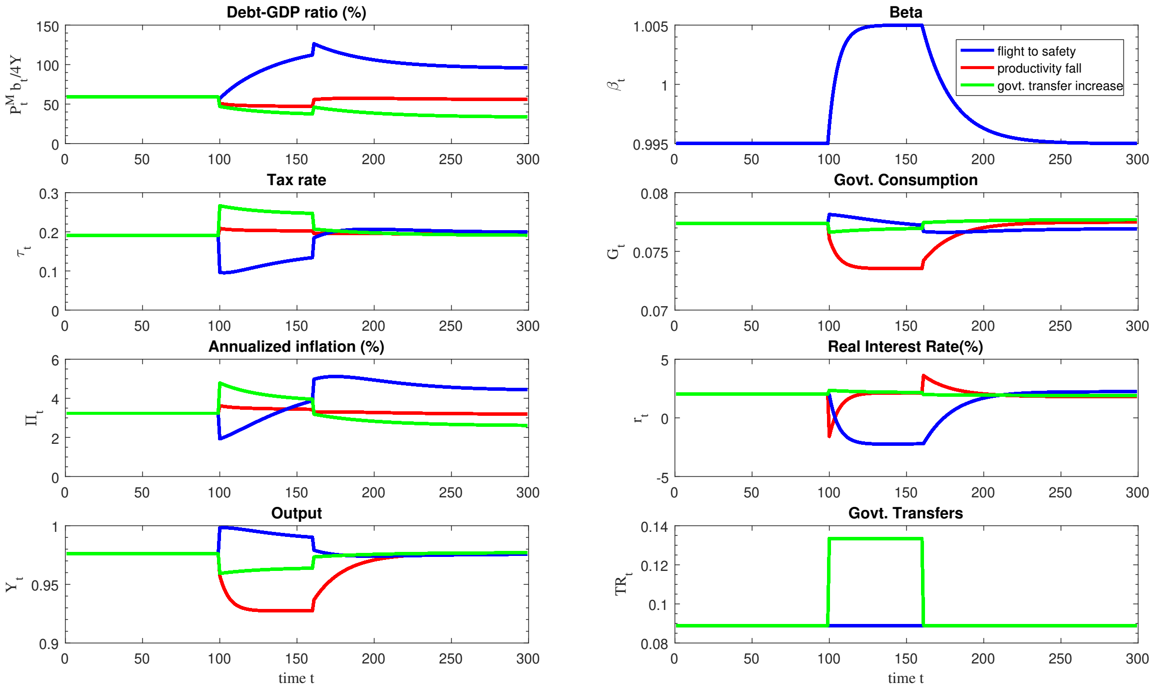

Figure 11: Impact of an Unexpected Crisis

and an increase in transfers should a crisis episode re-emerge, the new steady-state in normal times implies a debt-to-GDP ratio that is lower than previously. This reflects a precautionary desire on the part of the policymaker to reduce debt levels prior to a new crisis, thereby facilitating better outcomes should the crisis emerge. In contrast, in the case of a flight to safety on its own, this is actually good news as it reduces debt service costs, implying that it is less costly to sustain an inherited level of debt. This increases the debt-to-GDP ratio in normal times. However, inflation remains elevated as a result.

The impact of combining all these various shocks is then shown in Figure 12. Here, we see a fall in inflation (which is modest), output, and interest rates at the outset of the crisis, which prompts a fiscal stimulus that raises the debt-to-GDP ratio rapidly. After a while, the rising debt levels increase inflation, and fiscal policy is tightened to slow the rise in debt. There is a negligible tightening of monetary policy, although interest rates remain not far from the ZLB as a result of the ongoing flight to safety. Inflation continues to rise as debt increases, and when the crisis period ends, interest rates and output re-normalize. However, now that crisis episodes are known to be possible, and there is a perceived risk that they will re-occur, debt levels will not return to their original pre-crisis level. Instead, both debt and inflation remain high as policymakers anticipate future crises. The reason for this is that there is little desire to incur the short-run costs of debt reduction when policymakers know that there are likely to be sustained periods where a flight to safety reduces debt service costs. This is despite the fact that lowering debt would facilitate the use of fiscal policy for stabilization purposes when a future crisis hits.

The process also affects how the central bank responds to shocks. Figure 13 considers the marginal impact of a cost-push shock in normal vs flight-to-safety times, as well as with high or low levels of debt. The red (blue) lines imply that the shock occurs when debt levels are low (high), and the solid (dotted) line denotes a flight-to-safety (normal) regime, respectively. This indicates that the low debt stabilization bias during the flight-

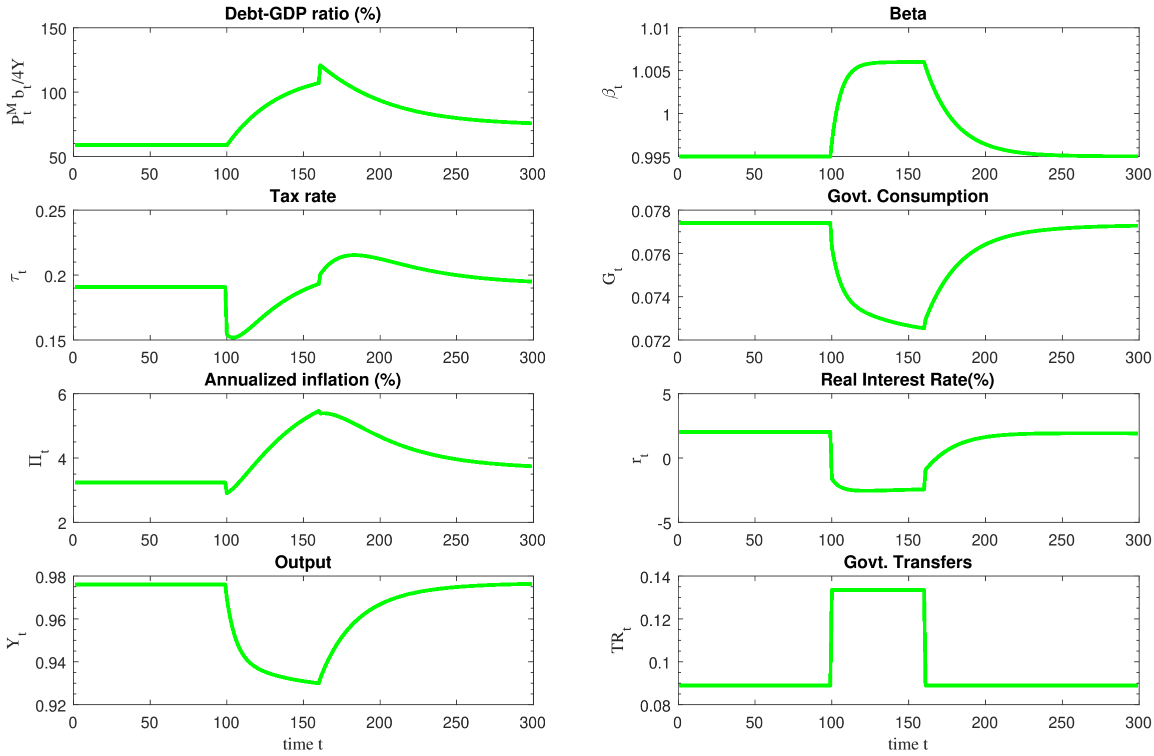

Figure 12: Impact of an Unexpected Crisis- All Factors

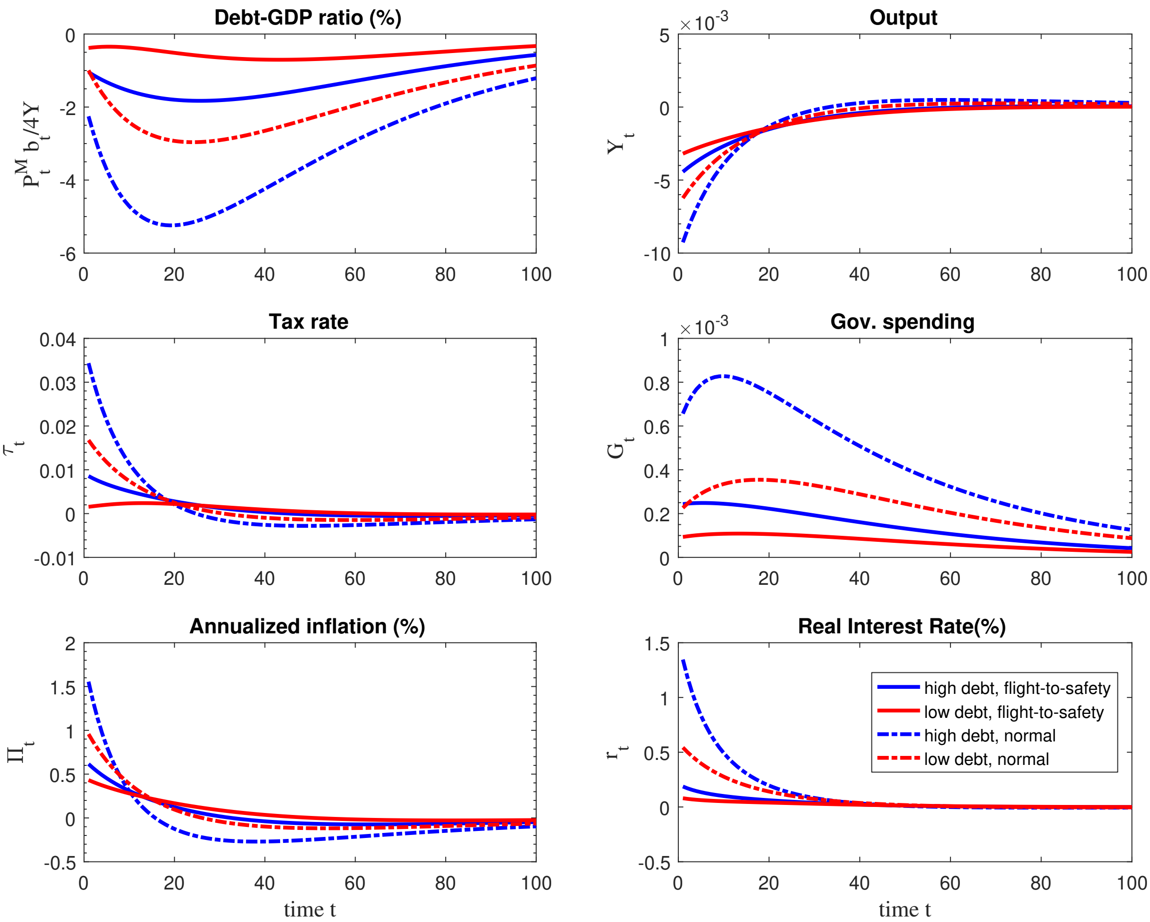

Figure 13: Differing responses to cost-push shock with high and low debt and/or a flight to safety

to-safety regime results in relatively low additional inflation from the cost-push shock, which further reduces output below its efficient level. In contrast, when we are in the normal regime, the cost-push shock is significantly more inflationary. When we turn to high levels of debt - the blue lines - the impacts are more pronounced. A cost-push shock occurring shortly after the re-normalization of interest rates (when debt levels are still high following the debt accumulation during the flight-to-safety episode) worsens an already large inflation bias problem fueling inflation by more than three times that observed under low debt levels. While the monetary policy response, in terms of real interest rates, rises with inflation, the ratio of the increase in inflation to the increase in real interest rates (consistent with the coefficient of a simple Taylor rule) falls significantly - in this sense, the high debt levels cause the central bank to moderate its anti-inflation response to cost-push shocks, although the Taylor principle would still be adhered to in observed data.

7 Conclusions

We utilized a new Keynesian model to explore the implications of central bank independence in an economy with a fiscal authority levying distortionary taxation to fund public consumption, transfers, and the interest costs of long-maturity debt. An inability to commit and the strategic interactions between these two players mean that, despite operational independence, the central bank is not free to meet its inflation target. The central bank faces an inflationary bias problem which is affected by fiscal policy and debt levels. In the short to medium term, making an inflation-averse central bank independent allows that bank to reduce inflation. However, at the same time, one of the constraints preventing the fiscal authority from issuing too much debt is the inflation bias that debt generates. When that cost is reduced as a result of central bank independence, the fiscal authority may raise debt levels significantly. This increase in debt worsens the inflation bias problem, and even with central bank independence, inflation will eventually rise above the level that would have occurred without independence. Central bank independence is still improving welfare - at the point of independence, the gains from reducing inflation outweigh any subsequent increase, while in the long run, reversing independence would be more costly than maintaining the status quo despite the increase in debt and inflation. It is important to stress that these results are not due to any unfavorable resolution of a game of chicken between the two policymakers - the central bank retains its operational independence throughout, and there is no weakening in its anti-inflation objectives as debt levels rise.

A crucial determinant of the impact of central bank independence is the relative myopia of both policymakers. While there are obvious political economy reasons to assume that elected politicians may act myopically, the situation for central bankers is less clear. They tend to focus on horizons over which the lags in the transmission mechanism play out, under the implicit assumption that they have control over inflation at that point. The current paper suggests that fiscally induced inflation biases do not give the central bank unfettered control over inflation even over a two to three-year horizon. Instead, policy outcomes are determined by the interplay between monetary and fiscal policies, and the extent to which the central bank looks beyond its usual planning horizon is crucial in determining the longer-term fiscal and inflation consequences of its current actions. A central bank focused on the near-term control of inflation is likely to experience a loss in such control as debt levels rise.

We then considered how variations in the natural interest rate may affect debt and inflation dynamics. The device of Markov switching in household discount factors was used to capture a temporary flight to safety, consistent with the fall in debt yields since the financial crisis. We showed, in a stylized example, that such a flight to safety could prompt a large fiscal stimulus and increase in debt at the same time as a fall in inflation. However, if the situation persists, inflation will eventually rise as debt levels rise. More significantly, as we exit the flight to safety and interest rates re-normalize, this would prompt a sharp increase in inflation and large fiscal consolidation in an attempt to reduce debt levels and offset the inflation bias they generate. This could be the situation we currently face.

We also explored how the response to shocks differs with high vs low debt, whether in normal times or during a flight to safety. During a flight to safety, cost-push shocks are less inflationary. However, the same shock occurring in normal times increases inflation by much more. Observed outcomes when debt levels are high are also consistent with the central bank moderating its response to shocks. This would appear as a decrease in the coefficient of a Taylor rule, although the Taylor principle would still be satisfied.

Finally, in order for the central bank to regain its control over inflation, we introduced a form of debt aversion on the part of the fiscal authority, which mirrors the inflation aversion of the central bank. We found that this was particularly effective in restoring the central bank's long-term control of inflation. A modest degree of debt aversion across a wide range of central bank inflation preferences allows the central bank to successfully reduce inflation without inducing the fiscal authority to accumulate additional debt, which would otherwise undermine the central bank's ability to control inflation.

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A Tables

Table 1: Parameterization

Parameter	Value	Definition
β	0.995	Quarterly discount factor (households)
$\beta^F = \beta^M$	0.982	Quarterly discount factor (policy maker)
σ	2	Relative risk aversion coefficient
σ^g	2	Relative risk aversion coefficient for government spending
φ	3	Inverse Frish elasticity of labor supply
ϵ	$14 + 1/3$	Elasticity of substitution between varieties
ρ	0.95	Debt maturity structure
χ	0.007	Scaling parameter associated with government spending
ρ_ϵ	0.95	AR-coefficient of cost push shock
σ_ϵ	0.01	Standard deviation of cost push shock
ϕ	50	Rotemberg adjustment cost coefficient

Table 2: Steady state values under the benchmark parameterization

Variable	Steady State	Definition
b	0.10	real long term debt
Y	0.97	output
$P^M b/(4Y) \times 100$	41%	debt-GDP ratio in terms of annual output
G/Y	7.8%	government spending
$(\Pi^4 - 1) \times 100$	3.2%	annualized inflation rate
τ	19%	income tax rate
i	5.2%	annualized nominal interest rate
r	2%	annualized real interest rate

B Technical Appendix (Not for Publication)

B.1 Summary of Model

We now summerise the model and its steady state before turning to the time-consistent policy problem.

Consumption Euler equation,

$$\beta_t R_t E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^\sigma \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1 \tag{41}$$

Pricing of longer-term bonds,

$$\beta_t E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^\sigma \left(\frac{P_t}{P_{t+1}} \right) \left(1 + \rho P^M_{t+1}\right) \right\} = P^M_t \tag{42}$$

Labor supply,

$$N_t^\varphi C_t^\sigma = (1 - \tau_t) \left(\frac{W_t}{P_t} \right) \equiv (1 - \tau_t) w_t$$

Resource constraint,

$$Y_t\left[1-\frac{\phi}{2}\left(\Pi_t-1\right)^2\right]=C_t+G_t \tag{43}$$

Phillips curve,

$$\begin{aligned}0=&(1-\epsilon_t)+\epsilon_t mc_t-\phi\Pi_t\left(\Pi_t-1\right)\\&+\phi\beta_t E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\right]\end{aligned} \tag{44}$$

Government budget constraint,

$$\begin{aligned}P_t^M b_t&=(1+\rho P_t^M)\frac{b_{t-1}}{\Pi_t}-\frac{W_t}{P_t}N_t\tau_t+G_t+Tr_t\\&=(1+\rho P_t^M)\frac{b_{t-1}}{\Pi_t}-\left(\frac{\tau_t}{1-\tau_t}\right)N_t^{1+\varphi}C_t^{\sigma}+G_t+Tr_t\\&=(1+\rho P_t^M)\frac{b_{t-1}}{\Pi_t}-\left(\frac{\tau_t}{1-\tau_t}\right)\left(\frac{Y_t}{A_t}\right)^{1+\varphi}C_t^{\sigma}+G_t+Tr_t\end{aligned} \tag{45}$$

Technology,

$$Y_t=A_tN_t \tag{46}$$

Marginal costs,

$$mc_t=W_t/\left(P_tA_t\right)=(1-\tau_t)^{-1}Y_t^{\varphi}C_t^{\sigma}A_t^{-1-\varphi}$$

The objective function for social welfare is given by,

$$E_0\sum_{t=0}^{\infty}\left(\prod_{i=-1}^{t-1}\beta_i\right)\left(\frac{C_t^{1-\sigma}}{1-\sigma}+\chi\frac{G_t^{1-\sigma_g}}{1-\sigma_g}-\frac{(N_t)^{1+\varphi}}{1+\varphi}\right) \tag{47}$$

There are two state variables in the baseline line model: debt b_t and the elasticity of substitution ϵ_t. When considering Markov switching between crisis and normal periods, we also allow for time variation in productivity, A_t and the households' time discount factor, β_t such that, alongside the state of being in a crisis or normal regime, these become state variables, too.

B.1.1 The Deterministic Steady State

Given the system of non-linear equations, the corresponding steady state system can be written as follows:

$$A=1$$

$$\frac{\beta R}{\Pi}=1$$

$$\frac{\beta}{\Pi}\left(1+\rho P^M\right)=P^M$$

$$(1-\tau)w=N^{\varphi}C^{\sigma}$$

$$Y\left[1-\frac{\phi}{2}\left(\Pi-1\right)^2\right]=C+G$$

$$(1-\epsilon_t)+\epsilon_t mc+\phi\left(\beta-1\right)\left[\Pi\left(\Pi-1\right)\right]=0$$
$$P^M b=(1+\rho P^M)\frac{b}{\Pi}-\left(\frac{\tau}{1-\tau}\right)Y^{1+\varphi}C^{\sigma}+G+Tr$$
$$Y=N$$
$$mc=w=(1-\tau)^{-1}Y^{\varphi}C^{\sigma}$$

which implies,

$$P^M=\frac{\beta}{\Pi-\beta\rho}$$
$$mc=w=\frac{\epsilon-1}{\epsilon}$$

$$\frac{C}{Y}=\left[(1-\tau)\left(\frac{\epsilon-1}{\epsilon}\right)\right]^{1/\sigma}Y^{-\frac{\varphi+\sigma}{\sigma}}$$
$$\frac{G}{Y}=1-\frac{C}{Y}=1-\left[(1-\tau)\left(\frac{\epsilon-1}{\epsilon}\right)\right]^{1/\sigma}Y^{-\frac{\varphi+\sigma}{\sigma}}$$
$$\frac{P^M b}{Y}=\frac{\beta}{1-\beta}\left[\tau\left(\frac{\epsilon-1}{\epsilon}\right)-\frac{G}{Y}-\frac{Tr}{Y}\right]$$

Note that,

$$Y^{\varphi+\sigma}\left(1-\frac{G}{Y}\right)^{\sigma}=(1-\tau)\left(\frac{\epsilon-1}{\epsilon}\right)\tag{48}$$

which will imply a level of output below that which would be chosen by a social planner, assuming taxes are non-negative.

B.2 Numerical Algorithm

This subsection describes the Chebyshev collocation method with time iteration used in the paper. See Judd (1998) for a textbook treatment of the numerical techniques involved.

Let $s_t=(b_{t-1},\beta_t,\epsilon_t)$ denote the state vector at time t, where real stock of debt b_{t-1} is endogenous, discount factor β_t and elasticity of substitution between goods ϵ_t are exogenous and respectively, with the following laws of motion:

$$P_t^M b_t=(1+\rho P_t^M)\frac{b_{t-1}}{\Pi_t}-w_t N_t\tau_t+G_t+tr_t;$$

$$\beta_t=\rho^{\gamma}\beta_{t-1}+(1-\rho^{\gamma})\,\beta^{\gamma}$$

with $\gamma=C,N$ respectively denoting the crisis and normal regime, $\beta^C>\beta^N$, the associated transition probability matrix governing the evolution of this two-state Markov process

$$\begin{bmatrix} p_N & 1-p_N \\ 1-p_C & p_C \end{bmatrix}\tag{49}$$

and p_γ denoting the probability of remaining in regime γ given currently in regime γ;

$$\ln(\epsilon_t)=(1-\rho_\epsilon)\ln(\epsilon)+\rho_\epsilon\ln(\epsilon_{t-1})+e_{\epsilon t},$$

with $0 \leq \rho_\epsilon < 1$ and $e_{\epsilon_t} \overset{i.i.d}{\sim} N(0, \sigma_\epsilon^2)$.

There are 7 endogenous variables and 6 Lagrangian multipliers in the optimal policy problem. It is convenient to consider β_t as an endogenous variable, real interest rate r_t and the value function V_t for social welfare in the numerical algorithm. Correspondingly, there are 16 functional equations associated with the 16 variables

$$\{C_t, Y_t, \Pi_t, b_t, \tau_t, G_t, P_t^M, \lambda_{1t}^m, \lambda_{2t}^m, \lambda_{3t}^m, \lambda_{1t}^f, \lambda_{2t}^f, \lambda_{3t}^f, \beta_t, r_t, V_t\}.$$

Let's define a new function $X : \mathbb{R}^3 \to \mathbb{R}^{16}$, in order to collect the policy functions of endogenous variables as follows:

$$X(s_t) = \Big(C_t(s_t), Y_t(s_t), \Pi_t(s_t), b_t(s_t), \tau_t(s_t), P_t^M(s_t), G_t(s_t), \lambda_{1t}^m(s_t), ..., \lambda_{3t}^f(s_t), \beta_t(s_t), r_t(s_t), V_t(s_t))\Big)$$

Given the specification of the function X, the equilibrium conditions can be written more compactly as,

$$\Gamma\left(s_t, X(s_t), E_t\left[Z\left(X(s_{t+1})\right)\right], E_t\left[Z_b\left(X(s_{t+1})\right)\right]\right) = 0$$

where $\Gamma : \mathbb{R}^{3+16+4+2} \to \mathbb{R}^{16}$ summarizes the full set of dynamic equilibrium relationships, and

$$Z\left(X(s_{t+1})\right) = \begin{bmatrix} Z_1\left(X(s_{t+1})\right) \\ Z_2\left(X(s_{t+1})\right) \\ Z_3\left(X(s_{t+1})\right) \\ Z_4\left(X(s_{t+1})\right) \end{bmatrix} \equiv \begin{bmatrix} M(b_t, \beta_{t+1}, \epsilon_{t+1}) \\ L(b_t, \beta_{t+1}, \epsilon_{t+1}) \\ (\Pi_{t+1})^{-1}\left(1+\rho P_{t+1}^M\right)\lambda_{3t+1}^m \\ (\Pi_{t+1})^{-1}\left(1+\rho P_{t+1}^M\right)\lambda_{3t+1}^f \end{bmatrix}$$

with

$$M\left(b_t, \beta_{t+1}, \epsilon_{t+1}\right) = \left(C_{t+1}\right)^{-\sigma} Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)$$
$$L\left(b_t, \beta_{t+1}, \epsilon_{t+1}\right) = \left(C_{t+1}\right)^{-\sigma}\left(\Pi_{t+1}\right)^{-1}\left(1+\rho P_{t+1}^M\right)$$

and

$$Z_b\left(X(s_{t+1})\right) = \begin{bmatrix} \frac{\partial Z_1(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_2(X(s_{t+1}))}{\partial b_t} \end{bmatrix} \equiv \begin{bmatrix} \frac{\partial M(b_t,\beta_{t+1},\epsilon_{t+1})}{\partial b_t} \\ \frac{\partial L(b_t,\beta_{t+1},\epsilon_{t+1})}{\partial b_t} \end{bmatrix}$$

More specifically,

$$L_1(b_t, \beta_{t+1}, \epsilon_{t+1}) = \frac{\partial\left[(C_{t+1})^{-\sigma}(\Pi_{t+1})^{-1}(1+\rho P_{t+1}^M)\right]}{\partial b_t}$$

$$= -\sigma(C_{t+1})^{-\sigma-1}(\Pi_{t+1})^{-1}(1+\rho P_{t+1}^M)\frac{\partial C_{t+1}}{\partial b_t}$$
$$-(C_{t+1})^{-\sigma}(\Pi_{t+1})^{-2}(1+\rho P_{t+1}^M)\frac{\partial \Pi_{t+1}}{\partial b_t} + \rho(C_{t+1})^{-\sigma}(\Pi_{t+1})^{-1}\frac{\partial P_{t+1}^M}{\partial b_t}$$

and

$$M_1(b_t, \beta_{t+1}, \epsilon_{t+1}) = \frac{\partial\left[\left(C_{t+1}\right)^{-\sigma} Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\right]}{\partial b_t}$$

$$= -\sigma(C_{t+1})^{-\sigma-1}Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial C_{t+1}}{\partial b_t} + \left(C_{t+1}\right)^{-\sigma}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial Y_{t+1}}{\partial b_t}$$
$$+ \left(C_{t+1}\right)^{-\sigma}Y_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial \Pi_{t+1}}{\partial b_t} + \left(C_{t+1}\right)^{-\sigma}Y_{t+1}\Pi_{t+1}\frac{\partial \Pi_{t+1}}{\partial b_t}$$

$$= -\sigma(C_{t+1})^{-\sigma-1}Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial C_{t+1}}{\partial b_t} + \left(C_{t+1}\right)^{-\sigma}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial Y_{t+1}}{\partial b_t}$$
$$+ \left(C_{t+1}\right)^{-\sigma}Y_{t+1}\left(2\Pi_{t+1}-1\right)\frac{\partial \Pi_{t+1}}{\partial b_t}$$

Note we are assuming $E_t\left[Z_b\left(X(s_{t+1})\right)\right] = \partial E_t\left[Z\left(X(s_{t+1})\right)\right]/b_t$, which is normally valid using the Interchange of Integration and Differentiation Theorem. Then, the problem is to find a vector-valued function X that Γ maps to the zero function. Projection methods, hence, can be used.

Following the notation convention in the literature, we simply use $s = (b, \beta, \epsilon)$ to denote the current state of the economy $s_t = (b_{t-1}, \beta_t, \epsilon_t)$, and s' to represent next period state that evolves according to the law of motion specified above. The Chebyshev collocation method with time iteration which we use to solve this nonlinear system can be described as follows:

1. Define the collocation nodes and the space of the approximating functions:

   - Choose an order of approximation (i.e., the polynomial degrees) n_b, n_β and n_ϵ for each dimension of the state space $s = (b, \beta, \epsilon)$, then there are $N_s = (n_b+1)\times(n_\beta+1)\times(n_\epsilon+1)$ nodes in the state space. Let $S = (S_1, S_2, ..., S_{N_s})$ denote the set of collocation nodes.
   - Compute the n_b+1, $n_\beta+1$ and $n_\epsilon+1$ roots of the Chebychev polynomial of order n_b+1, $n_\beta+1$ and $n_\epsilon+1$ as

$$z_b^i = \cos\left(\frac{(2i-1)\pi}{2(n_b+1)}\right), \text{ for } i = 1, 2, ..., n_b+1.$$
$$z_\beta^i = \cos\left(\frac{(2i-1)\pi}{2(n_\beta+1)}\right), \text{ for } i = 1, 2, ..., n_\beta+1.$$
$$z_\epsilon^i = \cos\left(\frac{(2i-1)\pi}{2(n_\epsilon+1)}\right), \text{ for } i = 1, 2, ..., n_\epsilon+1.$$

   - Compute collocation points β_i as

$$\beta_i = \frac{\overline{\beta}+\underline{\beta}}{2} + \frac{\overline{\beta}-\underline{\beta}}{2}z_\beta^i = \frac{\overline{\beta}-\underline{\beta}}{2}\left(z_\beta^i+1\right) + \underline{\beta}$$

   for $i = 1, 2, ..., n_\beta+1$, which map $[-1, 1]$ into $[\underline{\beta}, \overline{\beta}]$. Note that the number of collocation nodes is $n_\beta+1$. Similarly, compute collocation points b_i as

$$b_i = \frac{\overline{b}+\underline{b}}{2} + \frac{\overline{b}-\underline{b}}{2}z_b^i = \frac{\overline{b}-\underline{b}}{2}\left(z_b^i+1\right) + \underline{b}$$

   for $i = 1, 2, ..., n_b+1$, which map $[-1, 1]$ into $[\underline{b}, \overline{b}]$. Compute collocation points

ϵ_i as

$$\epsilon_i = \frac{\overline{\epsilon} + \underline{\epsilon}}{2} + \frac{\overline{\epsilon} - \underline{\epsilon}}{2} z_\epsilon^i = \frac{\overline{\epsilon} - \underline{\epsilon}}{2} \left(z_\epsilon^i + 1 \right) + \underline{\epsilon}$$

for $i = 1, 2, ..., n_\epsilon + 1$, which map $[-1, 1]$ into $[\underline{\epsilon}, \overline{\epsilon}]$. Note that

$$S = \{(b_i, \beta_j, \epsilon_k) \mid i = 1, 2, ..., n_b + 1,\ j = 1, 2, ..., n_\beta + 1, k = 1, 2, ..., n_\epsilon + 1\}$$

that is, the tensor grids, with $S_1 = (b_1, \beta_1, \epsilon_1)$, $S_2 = (b_1, \beta_1, \epsilon_2)$, ..., $S_{N_s} = \left(b_{n_b+1}, \beta_{n_\beta+1}, \epsilon_{n_\epsilon+1}\right)$.

- The space of the approximating functions, denoted as Ω, is a matrix of three-dimensional Chebyshev polynomials. More specifically,

$$\Omega(S) = \begin{vmatrix} \Omega(S_1) \\ \Omega(S_2) \\ \vdots \\ \Omega(S_{n_\epsilon+1}) \\ \vdots \\ \Omega(S_{N_s}) \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & T_0(\xi(b_1)T_0(\xi(\beta_1)T_1(\xi(\epsilon_1)) & T_0(\xi(b_1)T_0(\xi(\beta_1)T_2(\xi(\epsilon_1)) & \cdots & T_{n_b}(\xi(b_1)T_{n_\beta}(\xi(\beta_1)T_{n_\epsilon}(\xi(\epsilon_1)) \\ 1 & T_0(\xi(b_1)T_0(\xi(\beta_1)T_1(\xi(\epsilon_2)) & T_0(\xi(b_1)T_0(\xi(\beta_1)T_2(\xi(\epsilon_2)) & \cdots & T_{n_b}(\xi(b_1)T_{n_\beta}(\xi(\beta_1)T_{n_\epsilon}(\xi(\epsilon_2)) \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & T_0(\xi(b_1)T_0(\xi(\beta_1)T_1(\xi(\epsilon_{n_\epsilon+1})) & T_0(\xi(b_1)T_0(\xi(\beta_1)T_2(\xi(\epsilon_{n_\epsilon+1})) & \cdots & T_{n_b}(\xi(b_1)T_{n_\beta}(\xi(\beta_1)T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1})) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & T_0(\xi(b_{n_b+1})T_0(\xi(\beta_{n_\beta+1})T_1(\xi(\epsilon_{n_\epsilon+1})) & \vdots & \cdots & T_{n_b}(\xi(b_{n_b+1})T_{n_\beta}(\xi(\beta_{n_\beta+1})T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1})) \end{vmatrix}_{N_s \times N_s}$$

  where $\xi(x) = 2\left(x - \underline{x}\right) / \left(\overline{x} - \underline{x}\right) - 1$ maps the domain of $x \in [\underline{x}, \overline{x}]$ into $[-1, 1]$.

- Then for a given regime $\gamma = C, N$, at each node $s \in S$, policy functions $X_\gamma(s)$ are approximated by $X_\gamma(s) = \Omega(s)\Theta_X^\gamma$,
  where

$$\Theta_X^\gamma = \left[\theta_\gamma^c, \theta_\gamma^y, \theta_\gamma^\pi, \theta_\gamma^b, \theta_\gamma^\tau, \theta_\gamma^{\widetilde{p}}, \theta_\gamma^g, \theta_\gamma^{\lambda_1^m}, ..., \theta_\gamma^{\lambda_3^f}, \theta_\gamma^\beta, \theta_\gamma^r, \theta_\gamma^v\right]$$

  is a $N_s \times 16$ matrix of the approximating coefficients.

2. Formulate an initial guess for the approximating coefficients, $\Theta_X^{\gamma,0}$, and specify the stopping rule ϵ_{tol}, say, 10^{-6}.

3. For a given regime $\gamma = C, N$, at each iteration j, we can get an updated $\Theta_X^{\gamma,j}$ by implement the following time iteration step:

   - At each collocation node $s \in S$, compute the possible values of future policy functions $X_\gamma(s')$ for $k = 1, ..., q$. That is,

$$X_\gamma(s') = \Omega(s')\Theta_X^{\gamma,j-1}$$

     where q is the number of Gauss-Hermite quadrature nodes. Note that

$$\Omega(s') = T_{j_b}(\xi(b'))T_{j_\beta}(\xi(\beta'))T_{j_\epsilon}(\xi(\epsilon'))$$

     is a $q \times N_s$ matrix, with $b' = \widehat{b}(s; \theta_\gamma^b)$, $\beta' = \widehat{\beta}(s; \theta_\gamma^\beta)$, $\epsilon' = \rho_\epsilon \epsilon + z_k \sqrt{2\sigma_\epsilon^2}$, $j_b = 0, ..., n_b$, $j_\beta = 0, ..., n_\beta$, and $j_\epsilon = 0, ..., n_\epsilon$. The hat symbol indicates the corresponding approximate policy functions, so $\widehat{b}$ is the approximate policy for real debt, for example. Similarly, the two auxiliary functions can be calculated as follows:

$$M_\gamma(s') \approx \left(\widehat{C}(s';\theta^c_\gamma)\right)^{-\sigma} \widehat{Y}(s';\theta^y_\gamma)\widehat{\Pi}(s';\theta^\pi_\gamma)\left(\widehat{\Pi}(s';\theta^\pi_\gamma)-1\right)$$

and,

$$L_\gamma(s') \approx \left(\widehat{C}(s';\theta^c_\gamma)\right)^{-\sigma}\left(\widehat{\Pi}(s';\theta^\pi_\gamma)\right)^{-1}\left(1+\frac{\rho\widehat{P^M}\left(s';\theta^{\widetilde{p}}_\gamma\right)}{1-\rho\beta^\gamma}\right)$$

Note that we use $\widetilde{P}^M_t = (1-\rho\beta^\gamma)\,P^M_t$ rather than P^M_t in numerical analysis, since the former is far less sensitive to maturity structure variations.

- Now calculate the expectation terms $E\left[Z\left(X(s')\right)\right]$ at each node s. Let ω_k denote the weights for the quadrature, then

$$E\left[M_\gamma(s')\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\widehat{C}(s';\theta^c_\gamma)\right)^{-\sigma}\widehat{Y}(s';\theta^y_\gamma)\widehat{\Pi}(s';\theta^\pi_\gamma)\left(\widehat{\Pi}(s';\theta^\pi_\gamma)-1\right) \equiv \overline{M}_\gamma\left(s',q\right)$$

$$E\left[L_\gamma(s')\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\widehat{C}(s';\theta^c_\gamma)\right)^{-\sigma}\left(\widehat{\Pi}(s';\theta^\pi_\gamma)\right)^{-1}\left(1+\frac{\rho\widehat{P^M}\left(s';\theta^{\widetilde{p}}_\gamma\right)}{1-\rho\beta^\gamma}\right) \equiv \overline{L}_\gamma\left(s',q\right)$$

and for $\varsigma = m, f$

$$E_t\left[\left(\frac{1+\rho P^M_{t+1}}{\Pi_{t+1}}\right)\lambda^\varsigma_{3t+1}\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\frac{1+\frac{\rho\widehat{P^M}\left(s';\theta^{\widetilde{p}}_\gamma\right)}{1-\rho\beta^\gamma}}{\widehat{\Pi}(s';\theta^\pi_\gamma)}\right)\widehat{\lambda}^\varsigma_3(s';\theta^{\lambda^\varsigma_3}_\gamma) \equiv \Lambda^\varsigma_\gamma\left(s',q\right).$$

Hence,

$$E\left[Z\left(X(s')\right)\right] \approx E\left[\widehat{Z}\left(X(s')\right)\right] = \begin{bmatrix}\overline{M}_\gamma\left(s',q\right)\\ \overline{L}_\gamma\left(s',q\right)\\ \Lambda^\varsigma_\gamma\left(s',q\right)\end{bmatrix}$$

- Next calculate the partial derivatives under expectation $E\left[Z_b\left(X(s')\right)\right]$.

- Note that we only need to compute $\partial C_{t+1}/\partial b_t$, $\partial Y_{t+1}/\partial b_t$, $\partial \Pi_{t+1}/\partial b_t$ and $\partial P^M_{t+1}/\partial b_t$, which are given as follows:

$$\frac{\partial C_{t+1}}{\partial b_t} \approx \sum_{j_b=0}^{n_b}\sum_{j_\beta=0}^{n_\beta}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^c_{j_bj_\beta j_\epsilon}}{\overline{b}-\underline{b}}T'_{j_b}(\xi(b'))T_{j_\beta}(\xi(\beta'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \widehat{C}_b\left(s'\right)$$

$$\frac{\partial Y_{t+1}}{\partial b_t} \approx \sum_{j_b=0}^{n_b}\sum_{j_\beta=0}^{n_\beta}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^y_{j_bj_\beta j_\epsilon}}{\overline{b}-\underline{b}}T'_{j_b}(\xi(b'))T_{j_\beta}(\xi(\beta'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \widehat{Y}_b\left(s'\right)$$

$$\frac{\partial \Pi_{t+1}}{\partial b_t} \approx \sum_{j_b=0}^{n_b}\sum_{j_\beta=0}^{n_\beta}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^\pi_{j_bj_\beta j_\epsilon}}{\overline{b}-\underline{b}}T'_{j_b}(\xi(b'))T_{j_\beta}(\xi(\beta'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \widehat{\Pi}_b\left(s'\right)$$

$$\frac{\partial P^M_{t+1}}{\partial b_t} \approx \sum_{j_b=0}^{n_b}\sum_{j_\beta=0}^{n_\beta}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^{\widetilde{p}}_{j_bj_\beta j_\epsilon}}{\overline{b}-\underline{b}}T'_{j_b}(\xi(b'))T_{j_\beta}(\xi(\beta'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \widehat{P}^M_b\left(s'\right)$$

Hence, we can approximate the two partial derivatives under expectation

$$\frac{\partial E\left[M_\gamma(s')\right]}{\partial b}$$

$$\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left[\begin{array}{c} -\sigma\left(\widehat{C}(s';\theta_\gamma^c)\right)^{-\sigma-1}\widehat{Y}(s';\theta_\gamma^y)\widehat{\Pi}(s';\theta_\gamma^\pi)\left(\widehat{\Pi}(s';\theta_\gamma^\pi)-1\right)\widehat{C}_b\left(s'\right) \\ +\left(\widehat{C}(s';\theta_\gamma^c)\right)^{-\sigma}\widehat{\Pi}(s';\theta_\gamma^\pi)\left(\widehat{\Pi}(s';\theta_\gamma^\pi)-1\right)\widehat{Y}_b\left(s'\right) \\ +\left(\widehat{C}(s';\theta_\gamma^c)\right)^{-\sigma}\widehat{\Pi}(s';\theta_\gamma^\pi)\left(2\widehat{\Pi}(s';\theta_\gamma^\pi)-1\right)\widehat{\Pi}_b\left(s'\right) \end{array}\right]$$

$$\equiv \widehat{M}_{\gamma,b}\left(s',q\right),$$

$$\frac{\partial E\left[L_\gamma(s')\right]}{\partial b}$$

$$\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left[\begin{array}{c} -\sigma\left(\widehat{C}(s';\theta_\gamma^c)\right)^{-\sigma-1}\left(\widehat{\Pi}(s';\theta_\gamma^\pi)\right)^{-1}(1+\frac{\rho\widehat{P^M}\left(s';\theta_\gamma^{\widetilde{p}}\right)}{1-\rho\beta^\gamma})\widehat{C}_b\left(s'\right) \\ -\left(\widehat{C}(s';\theta_\gamma^c)\right)^{-\sigma}\left(\widehat{\Pi}(s';\theta_\gamma^\pi)\right)^{-2}(1+\frac{\rho\widehat{P^M}\left(s';\theta_\gamma^{\widetilde{p}}\right)}{1-\rho\beta^\gamma})\widehat{\Pi}_b\left(s'\right) \\ +\rho\left(\widehat{C}(s';\theta_\gamma^c)\right)^{-\sigma}\left(\widehat{\Pi}(s';\theta_\gamma^\pi)\right)^{-1}\widehat{P}_b^M\left(s'\right) \end{array}\right]$$

$$\equiv \widehat{L}_{\gamma,b}\left(s',q\right).$$

That is,

$$E\left[Z_b\left(X_\gamma(s')\right)\right]\approx E\left[\widehat{Z}_b\left(X_\gamma(s')\right)\right]=\left[\begin{array}{c}\widehat{M}_{\gamma,b}\left(s',q\right)\\ \widehat{L}_{\gamma,b}\left(s',q\right)\end{array}\right]$$

4. At each collocation node s, solve for $X_\gamma(s)$ such that

$$\Gamma\left(s,X_\gamma(s),E\left[\widehat{Z}\left(X_\gamma(s')\right)\right],E\left[\widehat{Z}_b\left(X_\gamma(s')\right)\right]\right)=0$$

   The equation solver *csolve* written by Christopher A. Sims is employed to solve the resulting system of nonlinear equations. With $X_\gamma(s)$ at hand, we can get the corresponding coefficient

$$\widehat{\Theta}_X^{\gamma,j}=\left(\Omega\left(S\right)^T\Omega\left(S\right)\right)^{-1}\Omega\left(S\right)^TX_\gamma(s)$$

5. Update the approximating coefficients for both regimes, $\Theta_X^{\gamma,j}=\eta\widehat{\Theta}_X^{\gamma,j}+(1-\eta)\,\Theta_X^{\gamma,j-1}$, where $0\leq\eta\leq1$ is some dampening parameter used for improving convergence.

6. Check the stopping rules. If $\left\|\Theta_X^{\gamma,j}-\Theta_X^{\gamma,j-1}\right\|<\epsilon_{tol}$ for both regimes, then stop, else update the approximation coefficients and go back to step 3.

When implementing the above algorithm, we start from lower-order Chebyshev polynomials and some reasonable initial guesses. Then, we increase the order of approximation and take the solution from the previous lower-order approximation as starting value. This informal homotopy continuation idea ensures us to find a solution.

Remark. Given the fact that the price P_t^M under each regime fluctuates significantly for larger ρ, in numerical analysis, we scale the rule for P_t^M by $(1-\rho\beta^\gamma)$, that is, $\widetilde{P}_t^M=(1-\rho\beta^\gamma)P_t^M$. In this way, the steady state of $\widetilde{P}_t^M$ is very close to β^γ, and $\widetilde{P}_t^M$ does not differ hugely as we change the maturity structure.

B.3 Debt Aversion

We consider the fiscal authority's problem after introducing a degree of debt aversion to their policy problem.

$$
\begin{aligned}
\mathcal{L}^f &= (1-\alpha_b)\left(\frac{C_t^{1-\sigma}}{1-\sigma}+\frac{\chi G_t^{1-\sigma_g}}{1-\sigma_g}-\frac{(Y_t)^{1+\varphi}}{1+\varphi}\right)-\frac{\alpha_b}{2}\left(\frac{\beta_t b_t C_t^{\sigma}E_t\left[L(b_t,\beta_{t+1},\epsilon_{t+1})\right]}{Y_t}\right)^2 \\
&+\beta E_t[V_{t+1}^f(b_t,\beta_{t+1},\epsilon_{t+1})] \\
&+\lambda_{1t}^f\left[Y_t\left(1-\frac{\phi}{2}\left(\Pi_t-1\right)^2\right)-C_t-G_t\right] \\
&+\lambda_{2t}^f\left[\begin{array}{c}(1-\epsilon_t)+\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi}C_t^{\sigma}-\phi\Pi_t\left(\Pi_t-1\right) \\ +\phi\beta_t C_t^{\sigma}Y_t^{-1}E_t\left[M(b_t,\beta_{t+1},\epsilon_{t+1})\right]\end{array}\right] \\
&+\lambda_{3t}^f\left[\begin{array}{c}\beta_t b_t C_t^{\sigma}E_t\left[L(b_t,\beta_{t+1},\epsilon_{t+1})\right]-\frac{b_{t-1}}{\Pi_t}\left(1+\rho\beta_t C_t^{\sigma}E_t\left[L(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right) \\ +\left(\frac{\tau_t}{1-\tau_t}\right)Y_t^{1+\varphi}C_t^{\sigma}-G_t-Tr_t\end{array}\right]
\end{aligned}
$$

Note we have used the definition of bond prices to eliminate bond prices from the debt aversion term in the government's objective function. We can now write the first-order conditions (FOCs) for the policy problem as follows:

Government spending,

$$(1-\alpha_b)\,\chi G_t^{-\sigma_g}-\lambda_{1t}^f-\lambda_{3t}^f=0 \tag{50}$$

which says that the government matches the marginal utility gain from higher government spending against the tightening of the resource constraint ($\lambda_{1t}^f \geq 0$), and government budget constraint ($\lambda_{3t}^f \geq 0$).

Output,

$$
\begin{aligned}
-(1-\alpha_b)\,Y_t^{\varphi}+\alpha_b\frac{\left(\beta_t b_t C_t^{\sigma}E_t\left[L(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right)^2}{Y_t^3}+\lambda_{1t}^f\left[1-\frac{\phi}{2}\left(\Pi_t-1\right)^2\right] \\
+\lambda_{2t}^f\left[\epsilon_t\varphi(1-\tau_t)^{-1}Y_t^{\varphi-1}C_t^{\sigma}-\phi\beta_t C_t^{\sigma}Y_t^{-2}E_t\left[M(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right] \\
+\lambda_{3t}^f\left[(1+\varphi)Y_t^{\varphi}C_t^{\sigma}\left(\frac{\tau_t}{1-\tau_t}\right)\right]=0
\end{aligned} \tag{51}
$$

such that the marginal costs of higher output - the reduction in utility due to the need to increase labor supply and the fuelling of inflation this implies,($\lambda_{2t}^f \leq 0$) - are equated to the marginal gains from relaxing the resource constraint ($\lambda_{1t}^f \geq 0$) (facilitating enhanced private and/or public consumption) and the fiscal benefits of increasing the tax base, ($\lambda_{3t}^f \geq 0$).

Taxation,

$$\lambda_{2t}^f\left[\epsilon_t(1-\tau_t)^{-2}Y_t^{\varphi}C_t^{\sigma}\right]+\lambda_{3t}^f\left[Y_t^{1+\varphi}C_t^{\sigma}(1-\tau_t)^{-2}\right]=0$$

simplifying,

$$\epsilon_t\lambda_{2t}^f+\lambda_{3t}^f Y_t=0 \tag{52}$$

which is the marginal condition capturing the fact that a higher (distortionary) tax rate increases marginal costs and fuels inflation t ($\lambda_{2t}^f \leq 0$), while at the same time generating tax revenues which relaxes the government's budget constraint ($\lambda_{3t}^f \geq 0$);

Inflation,

$$-\lambda_{1t}^{f}\left[Y_t\phi\left(\Pi_t-1\right)\right]-\lambda_{2t}^{f}\left[\phi\left(2\Pi_t-1\right)\right]$$
$$+\lambda_{3t}^{f}\left[\frac{b_{t-1}}{\Pi_t^2}\left(1+\rho\beta_t C_t^{\sigma}E_t\left[L(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right)\right]=0 \tag{53}$$

This condition exists since the monetary authority doesn't directly control inflation, implying that the fiscal authority also evaluates the inflationary consequences of its actions. The first line captures the conventional inflationary bias problem from the perspective of the fiscal authority. A higher inflation rate has direct resource costs ($\lambda_{1t}^{f}\geq 0$), but increases output through the NKPC, conditional on expectations at time t ($\lambda_{2t}^{f}\leq 0$). However, there is an additional benefit to inflation in that, again conditional on expectations, it deflates the real value of government debt, thereby relaxing the government's budget constraint ($\lambda_{3t}^{f}\geq 0$).

Government debt,

$$-\alpha_b\frac{\left(P_t^M\right)^2 b_t}{Y_t^2}-\alpha_b\frac{P_t^M b_t E_t\left[L_1(b_t,\beta_{t+1},\epsilon_{t+1})\right]\left(\beta_t b_t C_t^{\sigma}\right)}{Y_t^2}$$
$$+\beta^F\left[\frac{\partial E_t V_{t+1}^{f}(b_t,\beta_{t+1},\epsilon_{t+1})}{\partial b_t}\right]+\lambda_{2t}^{f}\left[\phi\beta_t C_t^{\sigma}Y_t^{-1}E_t\left[M_1(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right]$$
$$+\beta_t\lambda_{3t}^{f}\left[C_t^{\sigma}E_t\left[L(b_t,\beta_{t+1},\epsilon_{t+1})\right]+b_t C_t^{\sigma}E_t\left[L_1(b_t,\beta_{t+1},\epsilon_{t+1})\right]-\rho\frac{b_{t-1}}{\Pi_t}C_t^{\sigma}E_t\left[L_1(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right]=0$$

where

$$L_1(b_t,\beta_{t+1},\epsilon_{t+1})\equiv\partial L(b_t,\beta_{t+1},\epsilon_{t+1})/\partial b_t$$
$$M_1(b_t,\beta_{t+1},\epsilon_{t+1})\equiv\partial M(b_t,\beta_{t+1},\epsilon_{t+1})/\partial b_t$$

Note that by the envelope theorem,

$$\frac{\partial V_t^{f}(b_{t-1},\beta_t,\epsilon_t)}{\partial b_{t-1}}=-\frac{\lambda_{3t}^{f}}{\Pi_t}\left(1+\rho\beta_t C_t^{\sigma}E_t\left[L(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right)$$

the FOC for taxation (52), and the definition of bond prices (7), we can write the FOC for government debt as,

$$-\alpha_b\frac{\left(P_t^M\right)b_t}{Y_t^2}\left(P_t^M+E_t\left[L_1(b_t,\beta_{t+1},\epsilon_{t+1})\right]\beta_t b_t C_t^{\sigma}\right)$$
$$+P_t^M\lambda_{3t}^{f}-\beta^F E_t\left[\frac{\lambda_{3t+1}^{f}}{\Pi_{t+1}}\left(1+\rho P_{t+1}^M\right)\right]$$
$$-\beta_t\lambda_{3t}^{f}C_t^{\sigma}\left[\phi\epsilon_t^{-1}E_t\left[M_1(b_t,\beta_{t+1},\epsilon_{t+1})\right]-(b_t-\rho\frac{b_{t-1}}{\Pi_t})E_t\left[L_1(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right]=0 \tag{54}$$