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Monetary and financial policy with privately optimal risk taking.

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Monetary and financial policy with privately optimal risk taking *

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We present a model with macroprudential externalities emerging from market allocation of aggregate risks. The model predicts a paradox of safety: an increase in household risk aversion increases the volatility of output and consumption. Optimal monetary and macroprudential policies are designed to stabilise the economy whilst not exacerbating moral hazard in future periods. There is typically a macroprudential role for monetary policy, sometimes a dominant role, even when macroprudential policies are set optimally. But there are limits too. A monetary policy that focuses overly on financial stability loses control of inflation.

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Introduction

It is widely accepted that monetary policymakers should take financial conditions into account when making policy, despite the widespread powers now allocated to prudential regulatory authorities. What is less clear are the mechanisms through which policy responses to financial conditions support the dual mandate of monetary policy, whether financial conditions should be considered an end in an of themselves, and whether the maturing of prudential regulatory authorities will at some point eliminate the need for monetary policymakers to respond to financial conditions at all.

If monetary policies can improve risk-sharing, is the mechanism the replication of missing markets as in Bhandari et al. (2021), the revelation of information about macroeconomic states as in Caballero et al. (2024), or by changing microeconomic incentives as in the tradition of agency theory? Where monetary policymakers do respond to financial conditions, should this be solely in pursuit of a dual mandate of output and inflation stabilisation? Or should financial stability be an aim in its own right?

Limits to existing prudential policies are a common justification for monetary responses to financial conditions. To what extent are these limits a reflection of under-developed prudential frameworks, or of deep constraints constraining what can be achieved through prudential policy?[1]

Our answers to these questions are: Monetary policy should respond to financial conditions, even when prudential policy is constrained optimal and aggregate risk markets are complete. Accommodative policy following financial tightening helps the monetary policymaker achieve optimal policy aims. This is in part due to the inclusion of financial conditions in our welfare criterion, but also holds for a policy maker with a more standard dual mandate value function. Accommodative policy encourages entrepreneurs to accumulate more inside wealth than they would otherwise. The additional inside wealth improves contracting efficiency by reducing the payoff to moral hazard in downturns, which is when loan monitoring is particularly costly.

[1] See Kashyap and Siegert (2020) for a review of recent research and the evolution of views of policymakers views on these questions.

Overview

The core model is standard, with monopolistic competition in retail goods and either flexible or sticky prices. We add a financial friction to this core. Entrepreneurs each have a production technology where factor productivity has common and firm-specific stochastic components. Entrepreneurs seek outside financing, but their ability to share firm-specific risks with outside investors is limited by their hidden information about their own productivity. Entrepreneurs and households are free to trade securities contingent on aggregate risks, which are common knowledge.

The monitoring of firm-specific risk reports follows the imperfect state verification model of Duncan and Nolan (2019).[2] This is a useful framework for our purposes for three basic reasons. First, the model provides robust microfoundations for private debt contracts even when agents can increase and decrease their exposure to macroeconomic risks. Second, the model suggests that financial conditions follows both aggregate productivity shocks and risk/uncertainty shocks (aggregate risk markets notwithstanding). Third, it turns out that the model is straightforward to incorporate into the core macroeconomic model.

The extension adds one new equation (a law of motion for leverage) to the standard three equation macro model, and incorporates leverage in the Phillips curve, in the case of sticky prices, and in the IS curve. The approximate social welfare (loss) functions are also amended from the familiar loss function in intuitive ways to capture the dynamics of consumption inequality between households and entrepreneurs as well as the costs of financial risk bearing. In factor markets, entrepreneurs discount the marginal revenue products of factors due to the increased (firm-specific) risk that accompanies increased production. A wedge of inefficiency emerges between the marginal revenue products of capital and labor and their factor prices. This wedge enters the Phillips Curve in a similar way to working capital loan frictions (as in Jermann and Quadrini, 2012, for example). Entrepreneurs consume from their own wealth. Fluctuations in the distribution of wealth and credit frictions thereby generate an aggregate demand wedge in the IS curve that is similar to

[2]Duncan and Nolan (2019) presents an extension of Townsend (1979) that resolves the critiques of Border and Sobel (1987) and Mookherjee and Png (1989). The tractability of the model, along with the absence of any exogeneous limited liability constraints, supports the analysis of the implications macroeconomic risk sharing.

Cúrdia and Woodford (2016). Another closely related paper with similar relationships between credit frictions and the Phillips and IS curves is Sims et al. (2021).

There is good reason to believe that aggregate risk markets are open, at least to some extent, and that can have important implications for optimal outside finance contracts (Chari and Christiano, 2017). We allow for firms and households to allocate business cycle risk through market transactions. As a result, the model combines two sources of macroprudential externalities and subsequent motivations for intervention. First, as in Di Tella (2017) and Duncan and Nolan (2021), aggregate risk markets do not fully internalise the increased social costs of financial stress resulting from high leverage in downturns. Importantly, while this externality is limited to risk shocks in Di Tella (2017), it is present for technology and monetary policy shocks in our model. Second, as in Farhi and Werning (2016) and Schmitt-Grohe and Uribe (2012), aggregate risk markets do not fully internalise the aggregate demand externalities resulting from changes in the distribution of wealth.

The macroprudential externality in our model generates a *paradox of safety* (closely related to the *safetry trap* described by Caballero and Farhi, 2017). In canonical New Keynesian and Real Business Cycle models, higher household consumption risk aversion dampens business cycle fluctuations, both through consumption smoothing and through the wealth effect on labour supply. In our model, individual risk averse households, seeking to protect their wealth from downturns, buy safe financial assets. Entrepreneurs accept the other side of the trade, and aggregate risk is concentrated within firm balance sheets as a consequence. Paradoxically, higher risk aversion generates a further demand for safety, which in turn concentrates risk and ultimately increases volatility in hours, income and consumption.

Our analysis yields the following findings: First, optimal monetary policy should respond to financial conditions, even when macroprudential tools are available. Financial stress generates a trade-off similar to a New Keynesian cost-push shock. Second, the appropriateness of monetary and macroprudential policy responses depends on shock persistence. Accommodative monetary policy is best suited for responding to transitory shocks, where stimulus can be withdrawn as the shock dissipates. Macroprudential policy is more appropriate for persistent shocks. Relying on prudential policy to manage transitory shocks can generate undesirable persistent fluctuations in leverage and consumption inequality. Third, a monetary policy solely focused on maintaining financial stability can lead to unintended consequences. Specifically, it can generate permanently high inflation in response to temporary recessionary shocks. Prudential policies can complement monetary policy, especially in downturns, but cannot fully eliminate the inflationary consequences of a financial stability-focused monetary policy.

Related literature

The literature on this topic is large, diverse, and growing. Martin et al. (2021) and Laeven et al. (2022) are recent and insightful overviews. The topic became, of course, of immense interest following the financial crisis in 2008/9. Whatever the faults of monetary policies around the globe ahead of that crisis, economists were quick to identify and endorse the use of new instruments to counter systemic financial shocks. Allen and Rogoff (2011), for example, concluded that for some countries at least: "Controlling bubbles is a difficult task that needs as many tools as possible." The notion that monetary policy needs to be buttressed by macroprudential tools has become popular.

How these policies ought to be coordinated is still not settled. If macroprudential policy is efficient in addressing the relevant externalities, there is some indication that monetary policy should to a first approximation stick to its traditional objectives and not seek to help out as regards financial conditions (see Korinek and Simsek, 2016 and Caballero and Simsek, 2019). The underlying intuition of these and other contributions is that if macroprudential policies are able to knock out the externalities associated with systemic risk, then monetary policy ought to stabilise inflation or eliminate other nominal distortions. Of course, many contributors to this growing literature are aware that in practice macroprudential policy is unlikely to be fully effective, opening up the possibility of a systemic role for monetary policy not solely in addressing the impact of nominal rigidities but also with distortions associated with financial risks. On the other hand, some argue that the financial stability role of monetary policy is quite fundamental (Stein, 2012, 2013).

One closely related recent contribution is Caballero et al. (2024). They present a model with financial noise shocks that generate real fluctuations through trading behaviour. In their model, monetary policy, by responding to financial conditions, can help rational traders to identify the true aggregate state. This is an important distinction from our model. In our model, monetary policy, by responding to financial conditions, can help lender households to identify the microeconomic states of individual borrowers. This is the case because monetary policy that responds countercyclically to financial shocks induces borrowers to carry more equity forward into downturns, supporting loan monitoring and damping the costs of moral hazard. In their model, the central bank's ability to implement first-best allocations is limited by lags in the identification of and ability of the policymaker to respond to financial noise shocks. In our model, the central bank cannot implement first-best allocations as a result of the underlying information asymmetries; these can be moderated but not eliminated through policy.

The link between monitary policy and loan monitoring is related to contributions by Bhandari et al. (2021) and Sheedy (2014). They show how countercyclical monetary policy can improve risk sharing when agents are restricted to nominal contracts that are not contingent on aggregate states. In our model, agents are free to write contracts contingent on aggregate states, taking into account monetary policy responses, yet countercyclical monetary policy can still generate welfare gains, and still largely by helping to shore up debtors' net worth in downturns. This distinction is important for understanding how policy in our model works. In those papers, there is no conflict between policymakers and private agents; both want contracts to be contingent on aggregate states and the role of optimal policy is to replicate the "missing" markets for macroeconomic risk, tranferring wealth to debtors in downturns. In our model, these implicit wealth transfers can be traded away. Monetary policy stabilises financial conditions through prices, by increasing entrepreneurs' marginal value of equity carried forward into downturns.

Preview of results

In Section 2 we document the *paradox of safety* in our model (Proposition 1). When households have higher risk aversion, the economy is more volatile. Individual households' attempts to insure their consumption, by holding safe or countercyclical financial assets, concentrate risk in the firm sector. This amplifies the volatility of labour demand and production, increasing the volatility of consumption in equilibrium.

In Section 3 we characterise the optimal policy responses when households and entrepreneurs both share log utility, suspending the safety trap feature of our model. Monetary policy should accommodate financial stress, which generates a trade-off that is similar to a cost-push shock. Macroprudential policy should also respond to financial stress; if monetary policy is non-optimal, macroprudential policy should also respond to technology and demand shocks. Prudential policy generates medium run fluctuations in wealth and consumption inequality; when the welfare costs of these fluctuations are high, macroprudential interventions are smaller and monetary interventions are larger.

In Section 4 we relax the assumption of log utility for the worker household, reintroducing our paradox of safety, which generates financial amplification of technology shocks and monetary policy responses. We restrict our attention to monetary policy regimes that maintain zero anticipated inflation. Following persistent shocks, the short term financial stability benefits of accommodative monetary policy are reversed as the monetary authority restores target inflation. Accommodative monetary policy is best suited to respond to temporary technology shocks, where monetary stimulus can be withdrawn as the shock dissipates. Conversely, macroprudential policy is best suited to responding to persistent technology shocks. Macroprudential policy is not well suited to responding to temporary technology shocks, as its effects on firms' leverage and consumption inequality persist after the shock has dissipated.

In Section 5 we consider a monetary policymaker who seeks to maintain financial stability in all periods. This analysis follows Akinci et al. (2021), who study a comparable policy in a quantitative model based on Gertler and Kiyotaki (2010). In our model, a monetary policy that maintains financial stability generates permanently high inflation in response to a temporary recessionary shock.[3] Prudential policies can dampen but cannot eliminate the inflationary consequences of financial stability monetary policy.

[3]Akinci et al. (2021) also find that financial stability monetary policy is very accommodative to supply shocks.

Section 6 discusses why uncertainty shocks, despite being technology shocks and despite the reaction of macroprudential policy, typically also demand a robust monetary policy response. The final Section concludes.

1 The model

The model consists of a representative household, who supplies labour and capital to a large population of entrepreneurs who produce a common product with a risky productive technology. Entrepreneurs sell their produce to monopolistically competitive retailers owned by the representative household, who produce differentiated retail products for consumption by both the representative household and the entrepreneurs themselves. We start by describing the aggregate equilibrium conditions before turning to their derivation.

1.1 Equilibrium conditions

Let x_t denote real output, i_t the nominal interest rate, π_t the inflation rate, ξ_t an uncertainty shock, l_t firm leverage.[4] Each of these variables is expressed in terms of log deviation from their respective steady state levels. The principal equations of our model are Equations 1.1-1.3.

The IS curve

$$x_t = \mathbb{E}[x_{t+1}] - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t[\pi_{t+1}]\right) + \omega(1-\psi)\mathbb{E}_t\left[\Delta l_{t+1}\right] + \omega\psi(1-\rho_\xi)\xi_t, \quad (1.1)$$

The Phillips curve

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \lambda \mathrm{pp}_t, \quad (1.2)$$

The Leverage curve

$$l_t = \phi l_{t-1} + (1-\phi)\left(\omega\sigma\Delta\xi_t - \xi_{t-1} - \frac{\tilde{\sigma}}{\psi}\Delta x_t\right) - \delta_t, \quad (1.3)$$

where σ denotes the representative household's coefficient of relative risk aver-

[4]Without loss of generality, the uncertainty shock ξ_t is assumed to follow an AR(1) process $\xi_t = \rho_\xi\xi_{t-1} + \epsilon_{\xi t}$.

sion. In our model, household risk aversion σ is important in its own right as in the standard New Keynesian model for determining demand and labour supply responses to shocks. But, in addition, some dynamics in our model are driven by the difference between σ and the respective unit relative risk aversion of entrepreneurs. Where it is this difference between household and entrepreneurial risk aversion that is the key determinant of model relationships, we use the notation $\tilde{\sigma} := \sigma - 1$. The steady state entrepreneurial consumption share is denoted $\frac{\omega}{1+\omega}$, where ω is the ratio between household and entrepreneurial consumption in the steady state. ψ is the elasticity of the equity risk premium with respect to leverage and risk (see Appendix A.2). The composite parameter ϕ governs the persistence of leverage, and $(1 - \phi)$ is the elasticity of the equity risk premium with respect to the ratio of consumption marginal utilities. The operator Δ takes the growth rate of its argument, $\Delta l_t = l_t - l_{t-1}$. The following parameter definitions follow Galí (2008): σ denotes the households' CRRA coefficient, and β the households' time preference parameter. The composite parameter λ is defined as $\lambda := \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$ where θ is the Calvo parameter,[5] α the Cobb-Douglas labour share parameter, and ε is the demand elasticity of substitution between retail products.

The model consists of a population of identical households and a population of entrepreneurs (described further in Section 1.2). Leverage is a measure of the extent to which debt is used to boost (expected) output. We measure leverage as the ratio of expected entrepreneurial output divided by the opportunity cost of entrepreneurial wealth. After log-linearization, leverage is $l_t = x_t - c_t^e + \rho_t$, where c_t^e is entrepreneurial consumption and ρ_t is the equity risk premium. Given entrepreneurial wealth at the start of the period, higher expected output is the result of higher leverage—higher borrowing from the household sector. All output is consumed, therefore leverage and the equity risk premium, l_t and ρ_t, uniquely determine the distribution of consumption.

As in the benchmark New Keynesian model, the IS curve is derived from the households' intertemporal consumption plans. Expected household consumption growth is decreasing in the expected real interest rate, $\mathbb{E}_t[r_{t+1}] := i_t - \mathbb{E}_t[\pi_{t+1}]$.

[5]With apologies for the abuse of notation, we use θ to denote the Calvo parameter in the Phillips curve, and θ_t to denote an entrepreneur's within period individual specific shock. In each case, the notation is standard in the respective New Keynesian and Principal-Agent literatures.

Our IS curve is expressed in terms of output x, therefore we also need to take into account the expected growth of the household's consumption share of total output, $-\omega(1-\psi)\mathbb{E}_t\left[\Delta l_{t+1}\right] - \omega\psi(1-\rho_\xi)\xi_t$. In Appendix B, we present an alternative representation of aggregate demand, derived from taking a consumption weighted average over both households and entrepreneurs' intertemporal plans. This alternative representation shows that current period output is decreasing in both current period leverage and uncertainty, all else equal (see Equation B.17).

Leverage and uncertainty also affect the Phillips curve in our model, through their effect on wholesale producer prices, the prices paid in competitive markets for the homoegeneous intermediate good produced by entrepreneurs. As in the benchmark New Keynesian model, marginal labour costs are increasing in output x_t and decreasing in technology a_t.[6] Leverage and uncertainty affect both labour supply and labour demand for every level of the output gap. On the supply side, an increase in leverage increases the households' consumption share of output, reducing households' marginal utility and increasing wage demands for every level of the output gap (a wealth effect resulting from consumption inequality).[7] On the demand side, entrepreneurs hire labour before realising their individual specific productivity outturn. Each additional worker increases the risk of production outcomes to the entrepreneur, a risk that can only imperfectly be defrayed to outside investors. Increased leverage and/or uncertainty decrease the demand for labour for every level of the output gap (a labour wedge of inefficiency).[8]

$$\mathrm{pp}_t = \underbrace{(\tilde{\sigma}+\chi)\,x_t - \chi a_t}_{\text{benchmark model marginal costs}} + \underbrace{\sigma\omega(1-\psi)l_t - \sigma\omega\psi\xi_t}_{\text{consumption inequality wealth effect}} + \underbrace{\tau_t}_{\text{labour wedge}}, \tag{1.4}$$

where the labour wedge is increasing in both leverage and uncertainty,

$$\tau_t = \tau_l l_t + \tau_\xi \xi_t, \qquad \tau_l, \tau_\xi > 0, \tag{1.5}$$

[6] As is standard, we denote the inverse Frisch elasticity as φ, and the production Cobb-Douglas weight on labour as $1-\alpha$.

[7] Similarly, a decrease in uncertainty also increases the households' consumption share of output and increases wage demands for every level of the output gap.

[8] A similar labour wedge of inefficiency could be derived from working capital loan frictions (as in Jermann and Quadrini, 2012, for example).

and $\chi := \frac{1+\varphi}{1-\alpha}$, given φ, the households' inverse Frisch elasticity. After purchasing the wholesale good from entrepreneurs, retailers produce a differentiated good which is sold in monopolistically competitive markets subject to Calvo pricing frictions. For much of our analysis, we will combine the wealth effects and risk premia contributions of marginal costs into composite elasticities capturing both effects,

$$\mu_l = \sigma\omega(1-\psi) + \tau_l, \qquad \mu_\xi = -\sigma\omega\psi + \tau_\xi.$$

For our benchmark specification, both $\mu_l, \mu_\xi > 0$.[9]

Households and entrepreneurs can trade securities contingent on aggregate risks that are observed by all. In competitive equilibrium, aggregate risk sharing implies that consumption aggregates evolve according to Equation 1.6. Equation 1.6 resembles a standard risk sharing condition but for two additions. First, the equity risk premium ρ reflects a wedge between the growth of the marginal utility of the average entrepreneurial consumption bundle c_t^e, and the growth of average entrepreneurial marginal utility. This wedge results from incomplete risk sharing with respect to individual specific risks faced by entrepreneurs, and rises in response to increased leverage and uncertainty. Second, the macroprudential policy instrument δ_t acts to limit either population's exposure to aggregate risks:

$$\sigma\Delta c_t = \Delta c_t^e - \rho_t - (1+\sigma\omega(1-\psi))\delta_t \tag{1.6}$$

where the scaling factor $(1+\sigma\omega(1-\psi))$ serves for convenience. By Equation 1.6, we can forecast changes in the expected consumption share of output, despite aggregate risk sharing. When leverage l_t or uncertainty ξ_t increase, the risk borne by entrepreneurs increases, and the equity risk premium ρ_t will increase. This generates a wedge between the growth of expected entrepreneurial consumption and expected entrepreneurial marginal utility, and predictable fluctuations in the distribution of consumption. Ultimately, from 1.6 we can derive the leverage curve 1.3, which predicts a mean-reverting path of leverage over time, with leverage increas-

[9] In an alternative specification where factor market decisions are made after realising the entrepreneurs' individual specific productivity outturn, the risk premia contribution to marginal costs is zero, $\mu_l, \mu_\xi = 0$, and the resulting composite elasticities are positive with respect to leverage and negative with respect to uncertainty, $\mu_l, -\mu_\xi > 0$.

ing in uncertainty and decreasing in output. Aggregate risk markets imply that when household consumption falls, entrepreneurial consumption also falls. As a result, for $\sigma > 1$, a decrease in output will cause an increase in leverage (as in 1.3). In this way, aggregate risk sharing is a source of the financial amplification of shocks in our model.

The monetary policy instrument is the nominal interest rate i_t. Importantly, monetary policy only affects leverage when either household risk aversion differs from entrepreneurs' $\sigma \neq 1$, or in the presence of a macroprudential policy that responds to monetary policy or its effects. Otherwise, leverage is independent of monetary policy.

1.2 Derivation of the model

In this section we present the foundations of our model from which we derive the equilibrium conditions above. Our model consists of a household sector, which supplies labour and savings; a population of entrepreneurs, who produce a homogeneous wholesale good with a risky technology; a retail sector, which produces differentiated retail products from the wholesale good, and a policymaker with access to prudential and interest rate policy instruments.

1.2.1 Households

The representative worker household brings wealth q_t into period t, enjoys consumption c and dislikes labour hours n. They have the following value function, expressed recursively,

$$v(q_t) = \max_{z_t, c_t, n_t, q_{t+1}} \mathbb{E}_t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\varphi}}{1+\varphi} + \beta v(q_{t+1}) \right\}$$

Households' real wealth carried forward into period $t+1$, q_{t+1}, is the sum of the gross real return to their period t wealth $(1 + r_{t+1})q_t$, real labour income $w_t n_t$, and real profits remitted from retailers Π_t, less consumption c_t, plus the net returns from their trade in aggregate risk:

$$q_{t+1} = (1 + r_{t+1})q_t + w_t n_t + \Pi_t - c_t \underbrace{- \int_{s \in S} p_t(s) z_t(s_t) ds + z_t(s_{t+1})}_{\text{trade in aggregate risk}} .$$

Aggregate risk securities $z_t(s)$ are contingent on the aggregate state vector s. In our analysis s can include productivity shocks, uncertainty shocks, markup and cost push shocks, government purchases shocks, and monetary policy shocks. In practice, we consider this aggregate risk trade as a proxy for a wide range of financial decisions that shift agents' exposure to business cycle shocks, and shift risks between groups.[10] Wealth q_{t+1} is determined by decisions made in period t, but is contingent on time $t+1$ outcomes of exogeneous state variables, and is therefore measurable in the $t+1$ state-space.

1.2.2 Entrepreneurs

An entrepreneur's intertemporal problem can be described as follows:

$$v^e(q^e_t) = \max_{z^e_t, c^e_t, q^e_{t+1}} \mathbb{E}_{\Theta,t} \left\{ \log c^e_t + \beta^e v^e(q^e_{t+1}) \right\}$$

subject to

$$q^e_{t+1} = R_t(\theta_t, s_t) q^e_t - c^e_t - \int_{s \in S} p_t(s) z^e_t(s_t) ds + z^e_t(s_{t+1})$$

Superscript e denotes the entrepreneur, and $v^e(q^e)$ is the value function. $R(\theta, s)$ is the return to entrepreneurial wealth, q^e_t, and is the outcome of a privately optimal external finance contract, determined at the beginning of the period, and conditional on idiosyncratic states realised within the period. θ denotes the idiosyncratic state drawn from set Θ and privately known by the entrepreneur. Trade in aggregate risk markets is captured by the quantities $z^e(s)$, denoting the amount purchased of an

[10]The decision between mortgage fixed rate terms is an example. A longer fixed rate term will reduce the household's exposure to aggregate shocks that result in high interest rates, which would be harmful to households with short mortgage fixed rate terms. In this way, a longer mortgage fixed rate provides insurance against aggregate shocks that increase interest rates. This insurance doesn't remove risk from the aggregate economy, but it does shift the risk from the mortgage borrowing household to other agents who are happy to accept interest rate risk at an agreeable price.

asset with payoff 1 conditional upon the future state of the world being realised as state s. The current period price of this security is denoted $p(s)$. As indicated earlier, trade in securities indexed by the aggregate state are not hampered by any problem of asymmetric information; unlike idiosyncratic states, aggregate states are costlessly observed and verified by all agents. These markets are active.

At the start of period t, the aggregate state s_t is realised and the payoffs from aggregate risk securities $z^e_{t-1}(s_t)$ are paid/received. This leaves entrepreneurs with net wealth q^e_t. They combine this wealth with borrowed funds to hire capital goods and labour for production within period t. Importantly, entrepreneurs borrow and employ labour before realising their within period idiosyncratic productivity shock.

Entrepreneurs produce output according to the function,

$$f(k^e_t, n^e_t; a_t, \theta_t) = a_t \nu(\theta_t)(k^e_t)^\alpha (n^e_t)^{1-\alpha},$$

where ν maps the individual specific shock θ into productivity and a_t is an aggregate productivity shock. An individual entrepreneur hires labour and rents capital after observing the aggregate state a_t, but before observing their individual specific shock θ_t.

The optimality condition for entrepreneurs' labour hiring can be expressed as follows:

$$\frac{w_t}{\text{PP}_t}\mathbb{E}_{\Theta,t}\frac{1}{c^e_t(\theta_t)} = \mathbb{E}_{\Theta,t}\frac{f_{n^e}(k^e_t, n^e_t; a_t, \theta_t)}{c^e_t(\theta_t)},$$

where PP_t is the producer price.

The marginal product of labour and the marginal utility value of revenue to the entrepreneur both vary across states of the world. Entrepreneurs place a high marginal utility weight on revenue in (privately) bad states of the world, where $c^e(\theta)$ is low, and a low marginal utility weight on revenue in good states.[11]

$$\frac{w_t}{\text{PP}_t} = \mathbb{E}_{\Theta,t}\left[f_{n^e}(n^e_t;\theta_t)\right]\underbrace{\left(1 + \text{cov}_{\Theta,t}\left(\frac{f_{n^e}(n^e_t;\theta_t)}{\mathbb{E}_{\Theta,t} f_{n^e}(n^e_t;\theta_t)}, \frac{1/c^e_t(\theta_t)}{\mathbb{E}_{\Theta,t}\left[1/c^e_t(\theta_t)\right]}\right)\right)}_{:=1-\tau}$$

[11] Arellano et al. (2019) generate a similar labour wedge in a model with risk neutral entrepreneurs and agency costs based on Jensen (1986).

A positive covariance between entrepreneurs' individual specific productivity draws $\nu(\theta)$ and consumption marginal utility $1/c_t^e(\theta_t)$ generates a labour wedge τ between the average marginal revenue product of labour across entrepreneurs, and the wage rate. If entrepreneurs cannot defray all production risk to external financiers, then this labour wedge will be positive.

Entrepreneurs' homogeneous output is sold in competitive markets to retail firms, who produce differentiated retail consumption goods for sale to households and entrepreneurs in monopolistically competitive markets. Retailers are owned by the households, and face Calvo (1983) pricing rigidities. Their full problem is described in Appendix B.2.

1.2.3 Macroprudential policy

Macroprudential policy in our model influences the allocation of exposure to aggregate risks. Rather than introducing a specific instrument, we take a mechanism design approach to the information constraints faced by the macroprudential policymaker.

The setup here is isomorphic to a model where banks make risky (i.e., undiversifiable) loans to final goods firms. The high returns in downturns, associated with higher risks, would discourage such banks from insuring their balance sheets against recessionary risks. As a result, and as in the model above, leverage would be too high going into the downturn and banks' ability to lend in the downturn would be too low from a social perspective. In other words, there would be a macroprudential externality. In such a situation, policymakers acting optimally would seek to curtail risky lending and that could be implemented via Basel-type capital requirements and/or loan-to-value type restrictions (applied symmetrically across all banks). The approach we take here is for tractability and avoids explicit modelling of the banking sector, envisaging risk management restrictions directly between households and entrepreneurs.

Constraint 1 *Hidden storage. Entrepreneurs can hide wealth across periods at the market risk free real interest rate.*

Within periods, entrepreneurs can hide income and consumption from external creditors. Across periods, entrepreneurs can hide wealth from macroprudential policymakers. In the absence of hidden storage, entrepreneurs who hide income from external creditors would consume their hidden income within the period.[12]

Constraint 1 prohibits the policymaker from imposing different risk free interest rates for households and entrepreneurs. Savings are a complement to within period misreporting of income, so Constraint 1 eliminates a margin that the policymaker could use to dampen the costs of the within period moral hazard problem between entrepreneurs and their external financiers (Green and Oh, 1991). In our view, prudential policies that did attempt to impose different risk free interest rates across groups are unlikely to be implementable in practice, whereas prudential policies that focus on exposure to risk are more likely to be implementable.

It follows from Constraint 1 that, in expectation, both entrepreneurs' and households' expected growth of marginal utility are equated to the same discount rate. As a result, intertemporal risk sharing holds in expectation for any feasible prudential policy,

$$\sigma \mathbb{E}_t[\Delta c_{t+1}] = \mathbb{E}_t[\Delta c^e_{t+1}] - \mathbb{E}_t[\rho_{t+1}]. \tag{1.7}$$

Lemma 1 follows directly from (1.6,1.7).

Lemma 1 *The macroprudential wedge is unpredictable,* $\mathbb{E}_t[\delta_{t+1}] = 0$.

While the macroprudential wedge is unpredictable, this does not imply that the macroprudential policy tools are ex post responses to shocks. Ex post transfers between entrepreneurs and households, in isolation, would have no effect on allocations in our model; agents would be able to trade away these transfers in competitive markets for claims contingent on aggregate states. Rather, macroprudential policy is characterised by ex ante interventions that dampen or amplify the response of net wealth to unanticipated economic shocks. These interventions could take the form of regulations, including risk-based leverage limits.

Corollary 1 *Macroprudential policy dampens (or amplifies) the response of entrepreneurial net wealth to unanticipated fluctuations in income and uncertainty. Macroprudential policy does not affect the response of entrepreneurial net wealth to anticipated fluctuations in income and uncertainty.*

[12] Allowing storage across periods to be observable, as in Green and Oh (1991) and Khan and Ravikumar (2001), would likely generate further interesting policy tradeoffs.

The macroprudential policymaker in our model can prevent the deterioration of entrepreneurial balance sheets in a crisis, but cannot on its own recapitalise entrepreneurs after their balance sheets have deteriorated.

1.2.4 Welfare

In order to construct a measure of welfare, we weight the household and entrepreneurial populations using the Negishi (1960) method. Intuitively, our policymaker would not wish to transfer wealth between populations in the model's steady state. This ensures that policy interventions are motivated by efficiency.

We explicitly model entrepreneurs in our welfare function specifically for two reasons: First, entrepreneurs consume a significant and variable share of total output, and therefore entrepreneurial consumption contributes to the household consumption welfare losses from fluctuations in output. Second, any monetary and financial policy regime that explicitly harms entrepreneurs without generating a sufficient offsetting benefit for households is likely to be undesirable, and may interact negatively with the feasibility of other government policies that affect the distributions of income and consumption.[13]

Our quadratic loss function is described by (1.8). The first three terms are similar to the benchmark New Keynesian model, up to the adjustment for population weights $(1 + \omega)$.

$$\begin{aligned}2\Lambda = &(1+\omega)\frac{\varepsilon}{\lambda}\pi_t^2 + (1+\omega)\chi x_t(x_t - 2a_t) + \tilde{\sigma}x_t^2 \\ &+ \omega\left((1+\sigma\omega)(1-\psi)l_t + \tilde{\sigma}x_t\right)\left((1-\psi)l_t - \psi\xi_t\right) \\ &+ \omega l_t(\kappa_{ll}l_t + \kappa_{l\xi}\xi_t) + \text{t.i.p.}\end{aligned} \tag{1.8}$$

The parameters $\kappa_{ll}, \kappa_{l\xi}$ capture the convexity of the dispersion of consumption outturns with respect to individual specific productivity outturns. Inflation reduces the efficiency of labour hours due to increased price dispersion; high output or low aggregate productivity reduces the return to labour hours; log consumption volatility is costly for households, with those costs increasing in the degree of risk aversion.

[13] Note that when households have log utility, the inclusion of entrepreneurs' utility into the welfare function has no effect on optimal monetary policy, as monetary policy actions do not affect leverage and the distribution of consumption across households.

The second line captures volatility in the distribution of consumption between households and entrepreneurs and its effect on consumption welfare losses for households. The third line captures the welfare costs accruing to entrepreneurs as a result of individual specific dispersion in productivity outturns. The resulting welfare losses are convex in leverage, uncertainty and their interaction.

The special case of log household utility

When households enjoy consumption with log utility, the monetary policymaker's loss function collapses to the benchmark New Keynesian loss function (up to terms independent of policy, see Appendix C.5):

$$\Lambda_{\sigma=1} = \frac{1}{2}\left(\frac{\varepsilon}{\lambda}\pi_t^2 + \chi x_t(x_t - 2a_t)\right) + \text{t.i.p.}$$

From the perspective of the monetary policymaker, it is efficient for labour supply to not respond to fluctuations in the leverage and uncertainty and their associated labour wedge. This marks a departure from competitive optimisation, where households will supply labour based on market wages which are moderated by the labour wedge. The labour wedge itself reflects real inefficiencies resulting from costly and imperfect loan monitoring, and these costs are indeed part of the monetary policymaker's loss. But they don't appear in the policymaker's loss function as a consequence of macro risk sharing between agents. Under log utility, leverage and monitoring costs do not respond to fluctuations in employment or aggregate demand. For an individual entrepreneur-worker pair, an increase in hours worked increases levarage and associated monitoring costs. For the economy as a whole, an increase in hours worked also generates an increase in the net wealth of entrepreneurs under equilibrium risk sharing; as a result, monitoring costs remain unchanged. The wealth flows resulting from fluctuations in aggregate demand do impact dynamic consumption inequality, but the resulting welfare costs are second-order in magnitude, while the damping of fluctuations in monitoring costs and the labour wedge generates first order welfare benefits.

2 The paradox of safety

Caballero and Farhi (2017) introduced the concept of a *safety trap*: in an acute liquidity trap, households' efforts to eliminate risk work against them in equilibrium, exacerbating the shortage of safe assets and amplifying volatility in output. Our model has a similar property. Individual risk averse households seek protection from aggregate fluctuations through their financial asset holdings. Entrepreneurs take the other side of the trade, absorbing aggregate risk. This leaves risk concentrated among entrepreneurs, resulting in large procyclical fluctuations in entrepreneurial net wealth, and financial amplification of aggregate shocks.

An increase in household risk aversion can increase the volatility of output. Increased household risk aversion generates an increased demand for safe assets from households. In equilibrium, this further concentrates risk among entrepreneurs, increasing volatility in leverage. When risk aversion is high, a decrease in output causes a large increase in leverage, which in turn increases the marginal production costs (1.4), and reduces aggregate demand (1.1). For typical specifications of monetary policy, or in the flexible price version of the model, output further decrease as a consequence.

In the benchmark New Keynesian model, increased risk aversion encourages the smoothing of consumption over time, and increases the stabilising wealth effect of labour supply in response to fluctuations in output. In our model, these stabilising forces of increased risk aversion remain present, but are counteracted by the effect of risk aversion on the concentration of risk among entrepreneurs. Our result is formalised by Proposition 1.

Proposition 1 *The paradox of safety. Higher household risk aversion can increase the volatility of output.*

We can illustrate the paradox of safety by focusing on the flexible price version of the model with technology shocks only. We can then simplify the model to the following aggregate supply and leverage conditions:

$$0 = (\tilde{\sigma} + \chi)x_t - \chi a_t + \mu_l l_t$$

$$l_t = \phi l_{t-1} - (1-\phi)\frac{\tilde{\sigma}}{\psi}(x_t - x_{t-1})$$

A decrease in productivity a_t decreases output x_t. On impact, this also increases leverage, as under competitive macro risk sharing entrepreneurs' net wealth is sensitive to macroeconomic fluctuations. The increase in leverage increases the marginal production costs, and reduces output further. Combining the two conditions, we have

$$x_t = \upsilon_x x_{t-1} + \upsilon_a(a_t - \phi a_{t-1}).$$

The composite parameters υ_x, υ_a, and ϕ are all functions of household risk aversion σ, and are all increasing in σ for typical parameter values. Quantitatively, the most important parameter is υ_a, which captures the on-impact response of output to productivity shocks,

$$\upsilon_a = \frac{\chi}{\chi + \tilde{\sigma}\dfrac{1-\tau_l}{1+\sigma\omega(1-\psi)}}.$$

In the canonical New Keynesian model, the on-impact response of output to technology shocks under flexible prices is given by $\dfrac{\chi}{\chi+\tilde{\sigma}}$, and is unambiguously decreasing in risk aversion. In our model, risk aversion concentrates macro risk within entrepreneurs' balance sheets, amplifying the leverage response to shocks and generating a feedback loop further increasing supply costs. For typical parameterisations, the elasticity of the labour wedge with respect to leverage is greater than unity ($\tau_l > 1$) and as a result the sign flips from the New Keynesian benchmark, with higher risk aversion increasing the on-impact response to shocks.

The effect of household risk aversion σ on the persistence of fluctuations, at least as it contributes to unconditional variance in output $\mathbb{E}_0[x_t^2]$, is more muted; the effects of increases in both υ_x and ϕ can counteract each other. We provide a numerical example in Appendix E.4, Figure 6. We also provide a diagrammatic representation of the result in Appendix D.3, Figure 3.

3 Optimal policy under log utility

Household risk aversion is central to the financial amplification mechanism in our model, and restricting households to log utility means that fluctuations in leverage and the equity risk premium are solely the result of uncertainty shocks. We find this log utility benchmark to be a useful starting point for our analysis, before returning to the general model with greater household risk aversion in later sections.

In this section we present three optimal policy results. First, we characterise optimal macroprudential policy in a flexible price benchmark economy. Second, we characterise optimal monetary and macroprudential policy under sticky prices. Optimal monetary policy stimulates the economy following uncertainty shocks, which complements the optimal macroprudential response. Third, we characterise optimal macroprudential policy under an interest rate rule. In this case, we focus on technology shocks, where the interest rate rule does not optimally manage aggregate demand, creating a role for macroprudential policy that differs from the earlier regimes.

Assumption 1 *All of the results in this section only rely on the assumption that household utility is logarithmic,* $\sigma = 1$.

Assumption 1 is strong. It improves tractability at the cost of removing an important feedback mechanism from the model. Under log utility, the feedback from output to leverage (and financial stability) is broken. Net worth moves one-for-one with aggregate output. Broadly speaking, this assumption means that the monetary policy authority can treat leverage as exogenous. This makes the model very tractable and helps us identify costs and benefits of policy interventions. The financial sector does not amplify technology shocks under log utility, and all financial stress is the result of either uncertainty shocks, or of macroprudential policy. Macroprudential policy may optimally choose to generate a link between financial stability and technology shocks in order to dampen the output response to technology shocks. In Section 4 we analyse optimal policy responses to technology shocks with greater household risk aversion and financial amplification.

The full derivations for this section are available in Appendix F.

3.1 The flexible price benchmark

We start with a flexible price benchmark before re-introducing nominal rigidities and monetary policy. Appendix E derives the flexible price aggregate demand and supply equilibrium relationship,

$$\chi(x_t - a_t) = -\mu_l l_t - \mu_\xi \xi_t. \tag{3.1}$$

Real output is increasing with technology, but decreases with leverage and uncertainty shocks. Both leverage and uncertainty increase risk borne by entrepreneurs, reducing labour demand. In addition, an increase in leverage reflects an increase in household wealth, generating a negative wealth effect on labour supply. Holding all else equal, an increase in uncertainty increases the entrepreneurs' share of consumption, generating a positive wealth effect on labour supply which dampens the effect of uncertainty shocks on output.

By Lemma 1, the prudential policymaker is constrained to policies that satisfy $\mathbb{E}_t[\delta_{t+1}] = 0$. It is convenient to incorporate this constraint into the leverage curve (1.3) in order to arrive at the constraint specified by (3.2), expressed in terms of the feasible paths of leverage that the prudential policymaker can implement:

$$\mathbb{E}_t[\Delta l_{t+1}] = -(1-\phi)(l_t + \xi_t) + \omega(1-\phi)\mathbb{E}_t[\Delta \xi_{t+1}]. \tag{3.2}$$

The flexible price macroprudential policymaker's problem is described by Programme 1.

Programme 1

$$\min_{x,l} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[\begin{array}{l} (1+\omega)\chi\left(x_t^2 - 2x_t a_t\right) + \omega\left(\kappa_{ll} + (1+\sigma\omega)(1-\psi)^2\right) l_t^2 \\ +2\omega\left(\kappa_{l\xi} - (1+\sigma\omega)(1-\psi)\psi\right) l_t \xi_t \end{array} \right]$$

Subject to (3.1), (3.2).

Solving Programme 1 yields the following optimal macroprudential policy wedge:

$$\delta_t = \left(\frac{\omega\hat{\kappa}_{l\xi} + (1+\omega)\chi^{-1}\mu_l\mu_\xi}{\omega\hat{\kappa}_{ll} + (1+\omega)\chi^{-1}\mu_l^2} \left(\frac{\phi' - \phi}{\phi' - \rho_\xi} \right) - \frac{1 - \omega(\phi' - 1)}{\phi' - \rho_\xi} (1-\phi) \right) \epsilon_{\xi,t}, \tag{3.3}$$

where $\phi' = (\beta\phi)^{-1}$, the explosive eigenvalue associated with the shadow cost of the leverage constraint. The ratio

$$\frac{\omega\hat{\kappa}_{l\xi} + (1+\omega)\chi^{-1}\mu_l\mu_\xi}{\omega\hat{\kappa}_{ll} + (1+\omega)\chi^{-1}\mu_l^2}$$

is the current period marginal rate of transformation between the social costs of uncertainty and the social costs of leverage. Without loss of generality, $\hat{\kappa}_{l\xi}$ is the individual cost of increased entrepreneurial risk bearing resulting from greater covariance between leverage and uncertainty, and is weighted by the entrepreneurial Negishi weight ω. The product $\chi^{-1}\mu_l\mu_\xi$ captures the cost of reduced hours resulting from the labour demand and supply effects of leverage and uncertainty, which are particularly high when the labour margin is more elastic (when χ is small). The resulting costs are borne by all and are therefore Negishi weighted $(1+\omega)$. In sum, the numerator captures the extent to which a change in leverage can offset the marginal social costs of uncertainty, and the denominator captures the social costs of the resulting volatility of leverage. These relative costs are weighted by the relative persistences of uncertainty and leverage. If the persistence of leverage ϕ is high relative to the persistence of uncertainty ρ_ξ, then the policymaker will moderate their prudential response to uncertainty shocks.

In the competitive equilibrium, uncertainty shocks increase current period leverage but they reduce leverage over longer time horizons. When uncertainty is high, the return to inside wealth is also high, and entrepreneurs' inside wealth grows quickly. As leverage is persistent, macroprudential policy has a enduring effect on the path of leverage, and can exacerbate the medium term decrease in leverage in response to a contractionary uncertainty shock. This persistence may not be desirable. The second term,

$$-\frac{1-\omega(\phi'-1)}{\phi'-\rho_\xi}\,(1-\phi),$$

reflects the persistent effect of current period uncertainty on future leverage, and dampens the optimal macroprudential response to uncertainty shocks.

Optimal macroprudential policy does not respond to technology shocks in this economy. Under log utility, technology shocks do not generate fluctuations in leverage. The competitive allocation appropriately adjusts hours worked in response to changes in technology. We'll see in Section 3.3 that deviations from optimal aggregate demand management can generate a motivation for macroprudential policy even in the absence of feedback from output to leverage, and we'll see in Section 4 that when the representative household is more risk averse, financial amplification of technology shocks generates fluctuations in leverage and in turn motivates macroprudential policy.

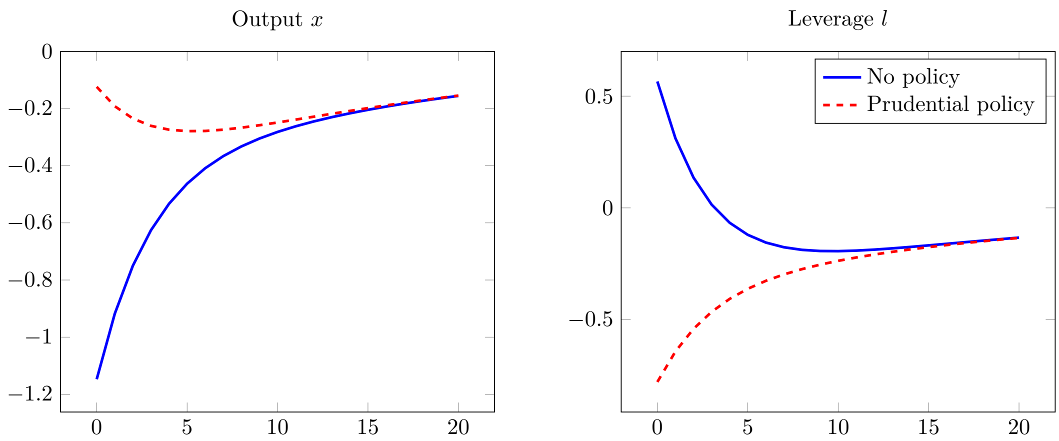

Figure 1: Responses to a recessionary uncertainty shock.

Figure 1 presents responses to a recessionary uncertainty shock, with and without macroprudential policy. In the absence of policy, entrepreneurial net wealth decreases sharply in response to the uncertainty shock, with leverage increasing as a result. The combination of high leverage and high uncertainty decreases labour demand, and output decreases in response. Under the optimal prudential policy, entrepreneurial net wealth is protected, and leverage decreases with output. Falling leverage helps to dampen the response of labour demand to the uncertainty shock, and as a result, the output response is dampened.

3.2 *Optimal monetary and prudential policy with nominal rigidities*

In this section we reintroduce nominal rigidities and solve for jointly optimal monetary and prudential policy under commitment. We separate the problem into two parts. Under log utility, the effect of the monetary policymaker's action on leverage is mediated through the optimal policy of the prudential policymaker. So, we first solve for the paths of output and inflation as functions of leverage, uncertainty and technology shocks—we interpret this as monetary policy—then we solve for the optimal path of leverage—we interpret this as the prudential policy.

The combined policymaker solves the following programme:

Programme 2 *Joint optimal monetary and prudential policy under log utility.*

$$\min_{\pi,x,l} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left((1+\omega) \left(\frac{\varepsilon}{\lambda} \pi_t^2 + \chi \left(x_t^2 - 2x_t a_t \right) \right) + \omega \hat{\kappa}_{ll} l_t^2 + 2\omega \hat{\kappa}_{l\xi} l_t \xi_t \right)$$

Subject to (3.2), and

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \lambda\chi x_t - \lambda\chi a_t + \lambda\mu_l l_t + \lambda\mu_\xi \xi_t.$$

The divine coincidence holds for technology shocks under log utility, so we focus our analysis on uncertainty shocks. Leverage and uncertainty enter the Phillips curve in a similar way to traditional New Keynesian cost-push shocks. Given the absence of feedback from monetary policy to leverage under log utility, optimal monetary policy faces similar trade-offs to monetary policy under cost-push shocks. Optimal inflation resembles a price-level targeting rule. Inflation increases in response to leverage and uncertainty, but eventually turns negative in order to restore the original price level (which is normalised to zero).

$$p_t = \varphi_1 p_{t-1} + \frac{\beta^{-1}\lambda}{\varphi_2 - \phi} \left(\mu_l l_t + \mu_\xi \left(1-\gamma\right) \xi_t \right). \tag{3.4}$$

where φ_1, φ_2 are the stable and explosive eigenvalues associated with optimal aggregate demand management familiar to New Keynesian models, and γ reflects the policymakers internalisation of expected effect of current uncertainty on future leverage.[14]

By allowing prices to increase on impact to recessionary uncertainty shocks, the

[14]

$$\varphi_1 = \frac{(1+\beta+\lambda\chi\varepsilon) - \sqrt{(1+\beta+\lambda\chi\varepsilon)^2 - 4\beta}}{2\beta}, \qquad \varphi_2 = \frac{1}{\beta\varphi_1},$$

$$\gamma = \frac{\phi - \rho_\xi + \frac{\mu_l}{\mu_\xi}(1+\omega(1-\rho_\xi))(1-\phi)}{\varphi_2 - \rho_\xi}, \qquad \lim_{\varepsilon\to\infty} \gamma = 0.$$

monetary policy authority bears a welfare cost from inflation but generates an increase in welfare by smoothing the path of output, consumption, and hours worked. This countercyclical monetary policy has no impact on leverage and risk bearing, with firms' net wealth increasing one for one with output to ensure that leverage remains invariant to monetary stimulus.

Optimal prudential policy is countercyclical, with the prudential policymaker lowering realised leverage in response to increases in the expected path of the current and future price level, and the risk bearing costs of uncertainty:

$$\delta_t = \frac{(1+\omega)\varepsilon\mu_l}{\omega\hat{\kappa}_{ll}}(\phi' - \phi)\sum_{j=0}^{\infty}(\beta\phi)^{j+1}(\mathbb{E}_t[p_{t+j}] - \mathbb{E}_{t-1}[p_{t+j}]) + \left(\frac{\hat{\kappa}_{l\xi}}{\hat{\kappa}_{ll}}\left(\frac{\phi' - \phi}{\phi' - \rho_\xi}\right) - \frac{1 - \omega(\phi' - 1)}{\phi' - \rho_\xi}(1 - \phi)\right)\epsilon_{\xi t}$$

Optimal monetary policy equates the marginal cost of inflation and output gaps resulting from uncertainty shocks. The prudential policymaker can therefore assess their optimal policy against the marginal impact of the policy on the economic costs of inflation. A unit decrease in leverage reduces marginal costs by μ_l on impact. Leverage propagates with persistence ϕ, so the effect of leverage on future prices decays at this rate. The welfare costs of future inflation are discounted at the social rate of time preference β. The Welfare costs of inflation are increasing in the elasticity of substitution ε and are borne by all consumers, assigned Negishi weight $1 + \omega$.

Equation 3.5 presents the optimal prudential policy in terms of the uncertainty shock alone. When the retail consumption elasticity of substitution approaches infinity, $\varepsilon \to \infty$, countercyclical monetary policy becomes prohibitively expensive, optimal inflation tends to zero and the optimal prudential policy collapses to the flexible price case (3.3). Conversely, as the retail consumption elasticity of substituion approaches zero, $\varepsilon \to 0$, countercyclical monetary policy can fully eliminate the social costs of uncertainty shocks that are transmitted through marginal costs, and prudential policy responds to the risk bearing and distributional costs of uncertainty and leverage only.

$$\delta_t = \left(\frac{\chi\omega\hat{\kappa}_{l\xi} + (1+\omega)\mu_l\mu_\xi\varsigma(1-\gamma)}{\chi\omega\hat{\kappa}_{ll} + (1+\omega)\mu_l^2\varsigma} \left(\frac{\phi' - \phi}{\phi' - \rho_\xi} \right) - \frac{1 - \omega(\phi' - 1)}{\phi' - \rho_\xi}(1 - \phi) \right) \epsilon_{\xi t} \tag{3.5}$$

where

$$\varsigma = \frac{\lambda\chi\varepsilon}{\beta} \frac{\phi'}{(\varphi_2 - \phi)(\phi' - \varphi_1)}, \qquad \lim_{\varepsilon \to 0} \varsigma = 0, \quad \lim_{\varepsilon \to \infty} \varsigma = 1.$$

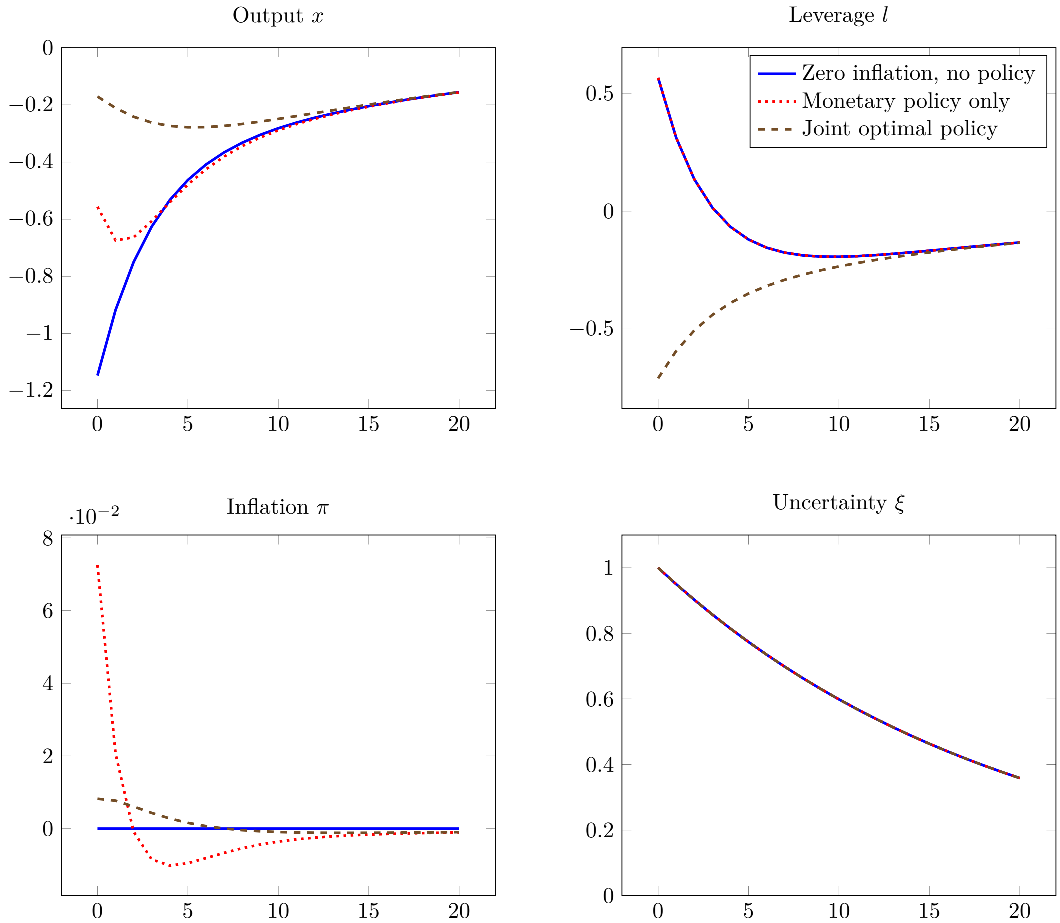

Figure 2: Monetary and prudential responses to a recessionary uncertainty shock.

Figure 2 presents the optimal monetary and joint optimal policy responses to a recessionary uncertainty shock, against the flexible price allocation in the absence of policy. The optimal monetary policy allows inflation to increase in the short run in response to the uncertainty shock, damping the output recession. Under log utility, there is no feedback from monetary policy to leverage. The optimal prudential response to the shock is slightly smaller under optimal monetary policy than under flexible prices (see Figure 1) but ultimately the response of output under optimal monetary and prudential policy is smaller than under prudential policy alone. In the absence of prudential policy, the optimal monetary policy allows for a large increase in inflation upon onset of the shock, and a subsequent period of low inflation to bring the price level back to its target. When prudential policy is optimal, the optimal inflation response to the uncertainty shock is dampened, and the subsequent overshooting of inflation is much smaller in magnitude. Optimal monetary policy still restores the original price level, but with much smaller deviations from zero inflation both at the onset of the shock and in subsequent periods.

3.3 Optimal prudential policy with an interest rate rule

In both the flexible price and optimal monetary policy regimes analysed above, there is no motivation for the prudential authority to respond to technology shocks, where flexible price or optimal monetary policy regimes can effectively manage the demand response to the technology shock, and where under log utility technology shocks do not affect financial stability.

When the aggregate demand response to technology shocks is non-optimal, there is a role for macroprudential policy to support aggregate demand management or reduce the costs of deviations from optimal aggregate demand management. This could be the case under a fixed exchange rate or monetary union regime, when monetary policy follows a simple Taylor-type interest rate rule, or optimises under discretion.[15] We focus on an interest rate rule, but present our result in terms of output and inflation elasticities to shocks, with the intention of facilitating a broader interpretation.

The macroprudential policy trade-offs in response to uncertainty shocks remain similar to the flexible price and optimal monetary policy cases. In order to avoid repetition, we remove uncertainty shocks from the model for this section, allowing technology shocks only.

[15] Chen, Kirsanova, and Leith (2017) show that US monetary policymaking is well characterised by an optimising monetary policymaker acting under discretion.

We assume the policy interest rate follows the simple rule

$$i_t = \phi_\pi \pi_t, \qquad \text{where} \;\; \phi_\pi > 1.$$

We then solve the system

$$x_t = \mathbb{E}[x_{t+1}] - (\phi_\pi \pi_t - \mathbb{E}_t[\pi_{t+1}]) - \sigma\omega(1-\psi)(1-\phi)l_t \tag{IS}$$

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \lambda\chi(x_t - a_t) + \lambda\mu_l l_t \tag{PC}$$

to arrive at a general solution with the form

$$x_t = \eta_{xa} a_t + \eta_{xl} l_t, \tag{3.6}$$

$$\pi_t = \eta_{\pi a} a_t + \eta_{\pi l} l_t, \tag{3.7}$$

where

$$\begin{aligned} \eta_{\pi a} &= -\frac{\lambda\chi}{(1-\beta\rho_a) + \frac{\phi_\pi - \rho_a}{1-\rho_a}\lambda\chi}, & \eta_{xa} &= -\left(\tfrac{\phi_\pi - \rho_a}{1-\rho_a}\right)\eta_{\pi a}, \\ \eta_{\pi l} &= \frac{\lambda\mu_l - \sigma\omega(1-\psi)\lambda\chi}{(1-\beta\phi) + \frac{\phi_\pi - \phi}{1-\phi}\lambda\chi}, & \eta_{xl} &= -\left(\tfrac{\phi_\pi - \phi}{1-\phi}\right)\eta_{\pi l} - \sigma\omega(1-\psi). \end{aligned} \tag{3.8}$$

Summaincreasing the above solution, both technology and leverage generate fluctuations in marginal costs, with elasticities χ and μ_l respectively. Leverage however also reduces aggregate demand, with elasticity $\sigma\omega(1-\psi)(1-\phi)$. The resulting decrease in output, represented by the term $-\sigma\omega(1-\psi)$ in the expression for η_{xl}, dampens or potentially reverses the response of marginal costs and inflation to fluctuations in leverage $(-\sigma\omega(1-\psi)\lambda\chi)$. The responses of inflation to technology and leverage fluctuations are decreasing in ϕ_π, but are increasing in the persistence of technology and leverage fluctuations respectively ρ_a and ϕ.

We impose the solution (3.6, 3.7) as a constraint on the macroprudential policymaker. The macroprudential policymaker then solves Programme 3.

Programme 3

$$\min_{\pi,x,l} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left((1+\omega) \left(\frac{\varepsilon}{\lambda}\pi_t^2 + \chi\left(x_t^2 - 2x_t a_t\right)\right) + \omega\hat{\kappa}_{ll} l_t^2 \right)$$

subject to (3.6), (3.7), and

$$\mathbb{E}_t[\Delta l_{t+1}] = -(1-\phi)l_t.$$

The optimal macroprudential wedge ω can be expressed as follows:

$$\delta_{t+1} = \left(\frac{\phi' - \phi}{\phi' - \rho_a}\right) \frac{(1+\omega)\left(\frac{\varepsilon}{\lambda}\eta_{\pi l}\eta_{\pi a} + \chi\eta_{xl}(\eta_{xa}-1)\right)}{\omega\hat{\kappa}_{ll} + (1+\omega)\left(\frac{\varepsilon}{\lambda}\eta_{\pi l}^2 + \chi\eta_{xl}^2\right)}\epsilon_{at+1}.$$

The sign of the prudential policy response to technology shocks is given by the sign of the following expression,

$$\frac{\varepsilon}{\lambda}\underset{-/+}{\eta_{\pi l}}\ \underset{-}{\eta_{\pi a}} + \chi\ \underset{-}{\eta_{xl}}\ (\underset{-}{\eta_{xa}} - 1).$$

Moving from right to left, under the interest rate rule, output increases in response to technology shocks ($\eta_{xa} > 0$) but not by enough to close the welfare relevant output gap ($\eta_{xa} - 1 < 0$). Output decreases in response to high leverage ($\eta_{xl} < 0$) and a decrease in output in response to high leverage is undesirable from the perspective of aggregate demand management (ie. there is no offsetting -1 attached to η_{xl}, as there is no output-leverage covariance term in the welfare function). The product $\eta_{xl}(\eta_{xa} - 1)$ is therefore positive: the prudential policymaker can reduce the costs of the insufficient output response to technology shocks by increasing ω, generating a countercyclical relationship between leverage and output, and introducing a financial amplification where there was none before.

This conclusion can change after taking into account the inflation costs of technology shocks. The inflation response to technology shocks is negative $\eta_{\pi a} < 0$. This reflects the insufficient response of output to technology shocks and the resulting counter-cyclical output gap. The inflation response to leverage, denoted as $\eta_{\pi l}$, can be either positive or negative. Leverage increases production marginal costs and reduces aggregate demand. If the demand response to leverage shocks is significant, then $\eta_{\pi l} < 0$. In this case, the prudential policymaker should introduce financial amplification of technology shocks. Otherwise, especially if the welfare costs of inflation fluctuations are high, the prudential policymaker should counteract technology shocks. This would generate a countercyclical path of leverage and reduce the elasticity of net wealth to output below one.

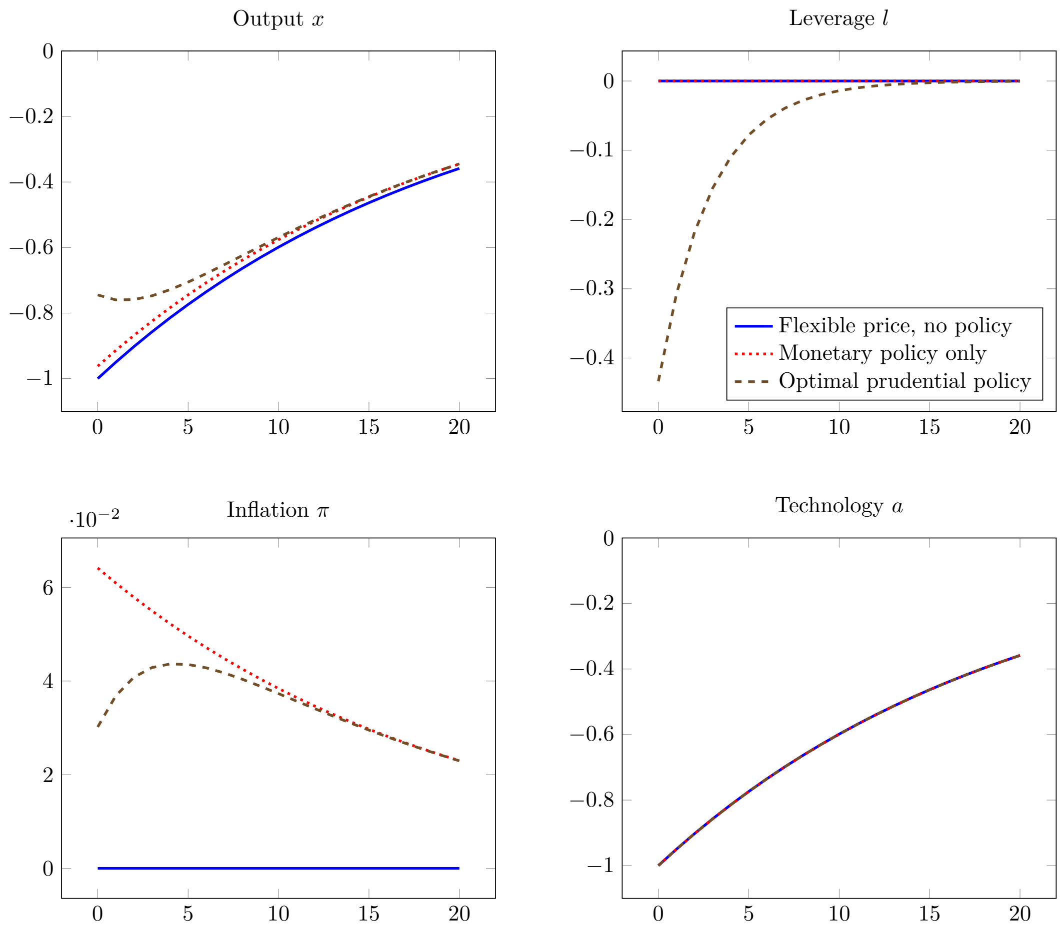

Figure 3: Prudential response to a recessionary technology shock ($\phi_\pi = 1.7$).

Figure 3 presents responses to a recessionary technology shock with and without prudential policy. Under log utility, the divine coincidence holds: optimal monetary policy maintains zero inflation in all periods and optimal prudential policy offers no response to technology shocks. We impose a Taylor-type simple rule with $i_t = \phi_\pi \pi_t$, where $\phi_\pi = 1.7$, a value that allows for a small positive output gap to emerge in response to a recessionary technology shock. Under this interest rate rule, the optimal prudential policy is countercyclical, further damping the output response to the shock relative to the flexible price (optimal policy) path. The countercyclical prudential policy dampens the response of entrepreneurial net wealth to the technology shock, with leverage decreasing in recessions and increasing in expansions. The decrease in leverage in response to the recessionary technology shock reduces marginal production costs, both by reducing the shadow monitoring costs of entrepreneurial loans, and by shifting some of the decrease in aggregate net wealth to the household sector, generating a wealth effect that increases labour supply. The decrease in marginal production costs reduces the inflation response to the shock, and thereby reduces the cost of the departure from optimal monetary policy.

4 Leaning against and cleaning up after financially amplified technology shocks

In Section 3 we showed that under log utility, the optimal aggregate demand response to technology shocks was to allow real output to rise and fall one-for-one with technology, in line with the canonical New Keynesian model under log utility. When households are less risk tolerant (ie. when their coefficient of relative risk aversion exceeds unity, $\sigma > 1$) any fall in output, even in response to a technology shock, generates a disproportionate fall in net wealth, concentrating risk among the entrepreneurs. This is a consequence of competitive risk sharing in our model—risk averse households seek to protect themselves from business cycles, but market clearing requires that the risk is borne by someone; entrepreneurs are willing to accept that risk at an agreeable price.

The concentration of risk on entrepreneurs balance sheets disproportionately reduces the net worth of entrepreneurs in response to contractionary technology or aggregate demand fluctuations, increasing leverage, and in turn increasing production costs.

This financial amplification effect can be derived from the leverage curve and the Phillips curve, as presented below:

$$\mathrm{pp}_t = (\tilde{\sigma} + \chi)\, x_t + \mu_l l_t + \mathcal{G}(a, \xi),$$

$$l_t = \phi l_{t-1} - (1-\phi)\frac{\tilde{\sigma}}{\psi}(x_t - x_{t-1}) + \mathcal{G}'(\xi, \delta),$$

In order to achieve a reduction in marginal costs of $(\tilde{\sigma}+\chi)$ in the current period, the monetary authority must reduce aggregate demand by m, where m is given by:

$$m := \frac{1}{1 - \underbrace{\frac{\mu_l}{\chi+\tilde{\sigma}}}_{\substack{\text{Elasticity of } \mathrm{pp}_t \\ \text{w.r.t. } l_t \text{ relative to } x_t}} \underbrace{\frac{(1-\phi)\tilde{\sigma}}{\psi}}_{\substack{\text{Elasticity of } l_t \\ \text{w.r.t. } x_t}}}.$$

In order to study how macroprudential policy should respond to financially amplified technology shocks, and how monetary policy could improve upon a strict inflation targeting policy, we simplify our benchmark model in a few important ways.

Assumption 2 *For our analysis in this Section, we make the following simplifying assumptions:*

(i) We remove uncertainty shocks from the model.

(ii) We restrict the monetary policymaker to pursue policies with zero expected future inflation. That is, we define γ as a policy parameter satisfying the following:

$$\mathbb{E}_t[\pi_{t+1}] = 0, \qquad \rightarrow \qquad \pi_t = \lambda\gamma\epsilon_{at}$$

Assumption 2 (i) removes uncertainty shocks, where the policymaker's problem is not fundamentally altered from the earlier cases studied under log utility in Section 3. Assumption 2 (ii) is very helpful for tractability and is a binding constraint on the monetary policymaker; the results that follow should be interpreted as helping us understand how a deviation from strict inflation targeting can improve outcomes, rather than as characterising optimal policy.

Following Assumption 2, we're left with the following Phillips curve and Leverage curve:

$$(\tilde{\sigma}+\chi)\, x_t = -\mu_l l_t + \chi a_t + \gamma\epsilon_{at}, \tag{4.1}$$

$$l_t = \phi l_{t-1} - (1-\phi)\tfrac{\tilde{\sigma}}{\psi}\Delta x_t - \delta\epsilon_{a,t}, \tag{4.2}$$

where $\chi = \frac{1+\varphi}{1-\alpha}$ and δ is the macroprudential policy parameter.

Consider a monetary policy that allows for a period of high inflation during recessionary technology shocks ($\gamma < 0$). From the equations above, in the current period, this generates a decrease in leverage relative to the counterfactual zero inflation policy. Lower leverage then feeds through to the Phillips curve (4.1), lowering marginal costs and increasing current period output.

Proposition 2 *Consider a monetary policy and a macroprudential policy that generate the same conditional response of output. For both interventions, leverage moves in the opposite direction to output on impact. The macroprudential policy intervention generates a larger (in absolute terms) leverage response on impact than the monetary policy intervention.*

Macroprudential interventions have a relatively large effect on current leverage and monetary policy interventions have a relatively large effect on current output. While both policy interventions have persistent effects on leverage and output, the persistence of monetary policy interventions is dampened, as all else equal higher output today leads to lower output growth tomorrow, increasing future leverage and offsetting the persistent decrease in leverage that would otherwise follow a period of expansionary monetary policy.

Departures from zero inflation incur a welfare cost to the monetary policy authority, which is not incurred as a result of macroprudential policy. However, the effects of monetary policy on output and leverage are different from those of macroprudential policy, and if a departure from zero inflation can reduce the expected welfare costs of volatility in output and leverage, then some departure from zero inflation will be optimal.

Proposition 3 *For generic parameterisations,*

i. both the monetary and macroprudential policy instruments should be used ($\gamma, \delta \neq 0$).

ii. in the absence of one instrument, the other instrument should be countercyclical ($\gamma, \delta < 0$).

Relative merits of each instrument

The welfare costs of technology shocks in the model, over and above the first-best welfare costs, primarily result from the feedback from output to leverage and back to output—the financial amplification in the model. Loosely speaking, policymakers seek to reduce this feedback, and the associated volatility of leverage, relative to the flexible price competitive allocation.

Equation 4.3 presents leverage as a function of past shocks. The equation separates the effects of the shock in the absence of policy (shock) from the effects of policy responses (monetary policy, prudential policy). The equation also separates the propagation of the on-impact shock from any anticipated reversal. For example, a one period recessionary shock reduces output growth and increases leverage today, but is followed by a predictable reversal, which increases output growth and reduces leverage in the following period. Similarly, given our assumption that anticipated inflation is zero, any expansionary monetary policy today is followed by a predictable reversal tomorrow.

$$\begin{aligned}
l_t = & -\frac{m-1}{\mu_l}\chi\left(\underbrace{\sum_{\tau=0}^{\infty}\phi^{\tau}\epsilon_{at-\tau}}_{\substack{\text{Endogenous}\\ \text{propagation}\\ \text{of shock}}} - \underbrace{(1-\rho_a)\sum_{\tau=1}^{\infty}\sum_{j=1}^{\tau}\phi^{j-1}\rho_a^{\tau-j}\epsilon_{at-\tau}}_{\substack{\text{Self-reversing of shock /}\\ \text{policy response}}}\right) && \text{(shock)}\\
& -\frac{m-1}{\mu_l}\gamma\left(\sum_{\tau=0}^{\infty}\phi^{\tau}\epsilon_{at-\tau} - \sum_{\tau=1}^{\infty}\phi^{\tau-1}\epsilon_{at-\tau}\right) && \text{(monetary policy)}\\
& - m\ \delta\sum_{\tau=0}^{\infty}\phi^{\tau}\epsilon_{at-\tau} && \text{(prudential policy)}
\end{aligned} \tag{4.3}$$

The monetary and macroprudential instruments differ in the persistence of their effects on the economy. The macroprudential instrument reduces the effect of the technology shock on leverage on impact, but its effect on leverage persists. The monetary policy instrument is largely self-reversing.[16] When technology shocks

[16]Mechanically, this is enforced by our restriction that forecast inflation is zero. In general, monetary policy can offset the leverage effects of persistent technology shocks at the cost of persistent inflation. See Section 5 for an example.

are persistent ($\rho_a \to 1$), the dynamics of the leverage response to macroprudential policy matches the dynamics of the leverage response to technology shocks. When technology shocks are not persistent ($\rho_a \to 0$) the dynamics of the leverage response to monetary policy shocks match the dynamics of the leverage response to technology shocks.[17] For this reason, macroprudential policies are relatively better suited to addressing the financial amplification of long term technology shocks and monetary policy is relatively better suited to addressing the financial amplification of short term technology shocks. Figure 4 presents an example to illustrate this result: when shocks are persistent, prudential policy dampens the path of leverage for the duration of the shock, while monetary policy only dampens the effect of the shock on impact, with leverage exceeding the no policy benchmark in subsequent periods. For iid. technology shocks, monetary policy dampens the response of leverage in all periods, while prudential policy causes an overshooting of the response of leverage from period 2 onwards.

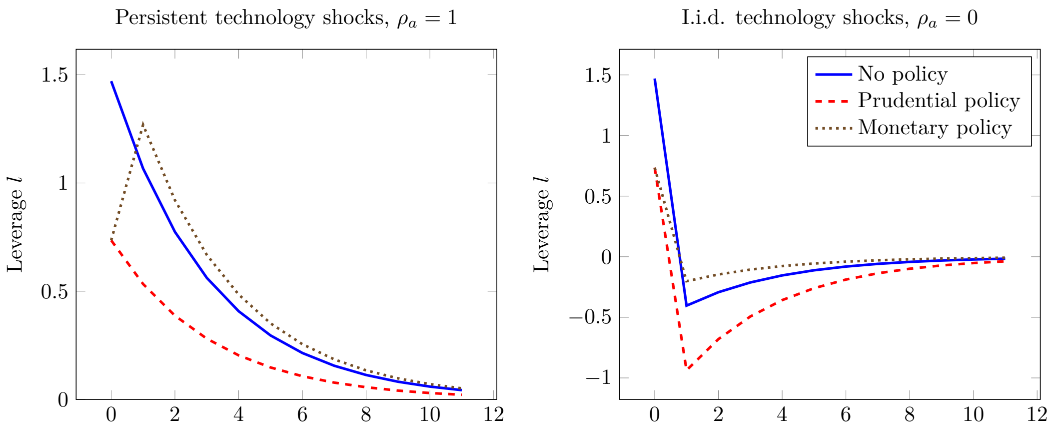

Figure 4: Responses to a unit recessionary technology shock.

Policy parameters γ, δ are chosen such that the period 1 response of leverage to the shock is constant across both policy tools.

Both policy tools share a common cost that their use for countercyclical stabilisation pushes hours worked above the level where their marginal contribution to total output does not compensate for their total sum of disutility and monitoring costs within the period. At the same time, both policy tools reduce the feedback from output to leverage, reducing the volatility of monitoring costs and thereby

[17] When verifying this from Equation 4.3, note that it is appropriate to set $\rho_a^0 = 1$ for $\rho_a = 0$.

increasing welfare. Monetary policy stabilisation suffers the additional cost of inflation, which is not a consequence of prudential stabilisation policy. Nevertheless, monetary policy is still part of the optimal policy mix. Importantly, the two policies have differential effects on leverage and output volatility, with prudential policy having a comparatively larger effect on leverage. Using both policy tools allows the policymaker to better address the marginal welfare costs of fluctuations in output and leverage. Optimal prudential policy alone leaves excess volatility in leverage and output that can be reduced by monetary policy at the second order welfare cost of temporary inflation. The greater the elasticity of substitution between goods ε, the higher the welfare costs of inflation, and comparitively the greater utility of countercyclical prudential policy relative to monetary policy. Similarly, the lower the elasticity of the labour margin (the higher is χ), the more inflation required to offset the output and leverage effects of a given change in technology, worsening the welfare costs of monetary policy stabilisation.

In short, policymakers should use both prudential policy and monetary policy to dampen fluctuations in leverage resulting from technology shocks. Policymakers should put greater reliance on prudential policy when technology shocks are persistent, the labour-output margin is inelastic, and the costs of inflation are high.

5 Financial stability interest rate policy

In this Section, we consider what monetary policy would be required in order to stabilise the equity risk premium ρ. The equity risk premium reflects the investment-savings wedge of inefficiency that characterises models of financial amplification of business cycle shocks.

Akinci et al. (2021) denote r^{**} to be the interest rate that stabilises financial frictions, and they study an interest rate policy maintaining r^{**} in a model based on Gertler and Kiyotaki (2010). Our model provides a tractable environment for studying this monetary policy strategy. Our analysis shares some similar insights to Akinci et al. (2021), but we also find a multiplicity of equilibria, where the economy can shift into high or low equilibria in response to temporary shocks. In our model, firms can anticipate future actions from the monetary policy authority, and adjust their risk taking behaviour today in response. Once the economy enters a high inflation, positive output gap equilibrium, financial stability can only be maintained through persistence of the positive output gap. The economy requires a departure from financial stability, for example a recessionary monetary policy shock, to bring inflation down to target and close the output gap. A financial stability interest rate policy prevents this adjustment from occuring, resulting in permanently higher or lower inflation.

Remark 1 *The equity risk premium ρ is stabilised when $l_t = -\xi_t$. without loss of generality, leverage decreases one-for-one with increased uncertainty.*

A proof of Remark 1, along with all derivations for this Section, is provided in Appendix H.

In order to stabilise the equity risk premium, our measure of financial stability, monetary policy must ensure that leverage moves inversely one-for-one with the uncertainty shock, $l_t = -\xi_t$. To achieve this, the monetary policy authority increases aggregate demand in response to uncertainty shocks, reducing entrepreneurial leverage and thereby negating the response of the equity risk premium to uncertainty shocks.

Combining Remark 1 with the leverage curve (1.3) yields the following condition that the financial stability interest rate must satisfy,

$$(1-\phi)\tfrac{\tilde{\sigma}}{\psi}\Delta x_t = \left(1 + (1-\phi)\sigma\omega\right)\Delta\xi_t - \delta_t. \tag{5.1}$$

It is important to emphasise that the ability of monetary policy policymakers to control leverage and the equity risk premium in our model is dependent on the financial amplification in the model, which results from households having a greater level of risk aversion than the entrepreneurs. This point is formalised by Remark 2.

Remark 2 *When the representative household has log utility ($\sigma = 1$), no financial stability interest rate policy exists.*

Proposition 4 provides a characterisation of the main properties of economic dynamics under a financial stability interest rate policy in the absence of prudential policy.

Proposition 4 *Characterisation of the financial stability interest rate in the absence of prudential policy.*

a. In response to technology shocks, the financial stability interest rate policy holds the real interest rate and output constant.

b. In response to an increase in uncertainty, the financial stability interest rate policy allows output to increase, and the real interest rate to fall. The effect on the nominal interest rate is ambiguous.

Technology shocks affect leverage and financial stability through their impact on real output and consumption. The financial stability interest rate policy maintains output and consumption at their steady state levels, preventing the deterioration of entrepreneurial net wealth that would otherwise occur during a recession. Inflation increases in response to recessionary technology shocks, and the nominal interest rate increases in line with inflation in order to stabilise the real interest rate.

An increase in uncertainty would typically be associated with an economic contraction in the flexible price model, or optimal monetary policies (see Section 3.2). Under the financial stability interest rate policy, the policymaker attempts to counteract the financial stability costs of increased uncertainty by reducing leverage, which requires an increase in output. The financial stability interest rate policy therefore allows output to increase in response to a (typically contractionary) increase in uncertainty. In order to generate this increase in output, the real interest rate must fall. But this fall in the real interest rate does not necessarily requite a decrease in the nominal interest rate. Uncertainty shocks generate a cost-push increase in inflation; for some parameter values, the financial stability nominal interest rate increases in response to an increase in uncertainty, and still accommodates an increase in real output.

Proposition 5 *Random walk property of the financial stability interest rate.*

a. Temporary departures from the financial stability interest rate (ie. monetary shocks) result in permanent output and inflation gaps.

b. Prudential policies, in combination with the financial stability interest rate, result in permanent output and inflation gaps.

The main problem with financial stability interest rate policy in or model is that if the economy is starting from a position with an output gap and non-target inflation, then there is no way to return to target inflation and eliminate the output gap. Using monetary policy to do so would require a temporary departure from financial stability.

Technology and uncertainty shocks on their own do not generate permanent departures from target inflation and output: in response to a technology shock, the financial stability interest rate maintains a zero output gap and steady state leverage; in response to an uncertainty shock, the stabilising path of leverage (and output) offsets the uncertainty shock, decaying at the same rate and ultimately bringing the economy back to the steady state.

Prudential policy throws the economy out of sync in response to uncertainty shocks. Prudential policy can affect the on-impact response of leverage to uncertainty shocks, but not the dynamic path of leverage. Ultimately, leverage returns to its steady state level before the output gap returns to zero, and at that point maintaining financial stability means maintaining an output gap and tolerating inflation.

5.1 The three interest rates: r^*, r^{**}, and the profit rate.

In response to technology shocks, under the financial stability interest rate policy both the real interest rate r_t^{**} and equity risk premium ρ_t are constant. Under strict inflation targeting, the real interest rate r_t^* and equity risk premium ρ_t can be expressed as follows, where $\mathcal{F}(\Omega_{t-1})$ denotes terms that are measurable in the period $t-1$ information set.

$$
\begin{aligned}
r_t^* &= -\frac{m\sigma'\chi}{\tilde{\sigma}+\chi}\left((1-\rho_a) + \frac{(1-\phi)^2\tilde{\sigma}}{\psi}\left(\underbrace{\frac{m\mu_l}{\tilde{\sigma}+\chi}}_{\text{cost push}} - \underbrace{\sigma\omega(1-\psi)}_{\text{demand pull}}\right)\right)a_t \\
&\quad + \mathcal{F}(\Omega_{t-1}) \\
\rho_t &= -\frac{(m-1)\chi\psi}{\mu_l}a_t + \mathcal{F}'(\Omega_{t-1})
\end{aligned}
$$

where $\frac{1}{\sigma'}$ is the population intertemporal elasticity of substitution, the weighted average of the intertemporal elasticities of substitution, weighted by the consumption shares of the households and entrepreneurs, adjusted for effect of consumption growth on the distribution of consumption:

$$\frac{1}{\sigma'} = \frac{1}{1+\omega(1-\psi)}\left(1 \cdot \frac{1}{\sigma} + \omega(1-\psi)\cdot 1\right).$$

It is helpful to contrast the real interest rate with that in the standard 3 equation New Keynesian model,

$$r^*_{NK,t} = -\frac{\sigma\chi}{\tilde{\sigma}+\chi}(1-\rho_a)a_t.$$

In our model, the financial amplification m implies that a larger real interest rate response is required to stabilise inflation following a given technology shock. In addition, the passthrough of fluctuations in output to leverage generates a cost-push effect on marginal costs, further amplifying the required real interest rate response to the shock, and a demand-pull effect dampening the required real interest rate response.

In summary, the inflation stabilising real interest rate responds more strongly to technology shocks in our model than in the standard 3-equation New Keynesian model when the financial accelerator is active, but this effect is moderated or could even be reversed when technology shocks and leverage are persistent and the effect of leverage on aggregate demand is large (that is, when the entrepreneurial share of consumption ω is large).

The equity risk premium ρ also increases on impact in response to a contractionary technology shock, with the response of the equity risk premium increasing in the magnitude of the financial amplification, the sensitivity of leverage to output and the elasticity of the equity risk premium with respect to leverage ψ. Ultimately, the profit rate $r^*_t + \rho_t$ is likely to increase significantly in response to contractionary technology shocks under strict inflation targeting, while remaining constant under the financial stability interest rate policy.

6 (Why) are uncertain recessions really inefficient?

Recessions driven by microeconomic uncertainty shocks are important for explaining macroeconomic fluctuations (Arellano, Bai, and Kehoe, 2019; Bloom et al., 2018; Christiano, Motto, and Rostagno, 2014; Di Tella, 2017). Our model shares features with the aforementioned papers that generate a useful role for uncertainty shocks in explaining business cycle outcomes. In particular, uncertainty shocks generate a reduction in aggregate demand, and a labour wedge of inefficiency, reducing the demand for labour below its marginal revenue product.

Uncertainty shocks generate a trade-off similar to that posed by New Keynesian cost-push shocks (Section 3.2). A monetary policymaker optimising under timeless commitment is willing to tolerate temporary inflation in order to dampen volatility in hours and output. Any deviation of output resulting from uncertainty shocks generates a welfare cost to the monetary policymaker, and their response to those deviations is only dampened by the costs of inflation.

This should be surprising. Uncertainty shocks are shocks to the technology process in the model. Why should the efficient level of hours worked be the same when microeconomic uncertainty is high, relative to when microeconomic uncertainty is low? When microeconomic uncertainty is high, financial stress and risk sharing is more costly than before. Production is really uncertain; uncertainty shocks are real shocks. If hours worked and production shouldn't decrease during an uncertainty shock recession, why do they decrease in the competitive equilibrium even in the absence of nominal rigidities?

Static and dynamic leverage constraints

At the start of period t, entrepreneurial leverage can be expressed as

$$l_t = x_t - q_t^e - i_{t-1} + \pi_t, \tag{6.1}$$

where q_t^e is the net wealth brought into period t by the entrepreneur. If we consider a monetary policy that increases output and inflation in the current period, this policy increases leverage for every level of net wealth. Higher leverage means a greater concentration of risk among individual entrepreneurs, and larger wedges of inefficiency in labour and capital markets.

If Equation 6.1 were the constraint faced in period t by the monetary policymaker, then monetary stimulus during uncertainty shocks would be counterproductive. Responding to an increase in uncertainty with an increase in leverage would just amplify the volatility of monitoring costs.[18]

Entrepreneurial net wealth brought into the period q_t^e depends on the anticipated monetary policy response to period t shocks. If a central bank pursues countercylical monetary policy in response to uncertainty shocks, then low interest rates during uncertain recessions will increase the entrepreneurial wealth brought into the period. The dynamic leverage constraint faced by the monetary policymaker is as follows (adapted from Equation 1.3):

$$l_t = \phi l_{t-1} + (1-\phi)\left(\omega\sigma\Delta\xi_t - \xi_{t-1}\right) - (1-\phi)\frac{\tilde{\sigma}}{\psi}\Delta x_t.$$

In the dynamic setting, stimulative monetary policy increasing output reduces leverage within the period. The sign of the leverage response to aggregate demand stimulus is reversed from the static constraint (6.1). Anticipated monetary stimulus in response to uncertainty shocks can dampen the concentration of risk through an increase in net wealth, reducing the costs of risk bearing and their effects on factor markets.

Monetary accommodation of uncertainty shocks and moral hazard

In the absence of nominal rigidities, our flexible price model is an Arnott-Greenwald-Stiglitz environment (Arnott and Stiglitz, 1986; Greenwald and Stiglitz, 1986). There is an information asymmetry between borrowers and lenders, with competitive anonymous trade in other markets. We have a clear theory of the role of government intervention in these environments: government policy should seek to discourage the complements of moral hazard.

When uncertainty is high, the cost of moral hazard is high. If businesses entered really uncertain business cycles with more equity—more skin in the game—then the cost of really uncertain business cycles would decrease. Monetary stimulus during

[18]It can be shown that in a one-period version of the model, the competitive allocation is constrained efficient.

high uncertainty restores equity values, discouraging moral hazard and reducing its effect on employment and output.

Uncertainty increases moral hazard and makes contracting more difficult. This drives wedges between savings and investment and between labour and production. While uncertainty shocks are real shocks to the technological process, they should still be addressed (at least in part) by accommodative monetary policy.

7 Discussion

Throughout this project, we had many discussions about how to construct the markets-closed counterfactual to our benchmark economy which has open aggregate risk markets. Would we impose fixed one-period nominal bonds, inflation protected one period bonds, multi-period fixed rate bonds, "floating rate" bonds with an interest rate determined in the following period? This decision is important for any counterfactual exercise, because each of the above choices affect the distribution of gains and losses in response to aggregate shocks. At some point, we realised that by-and-large, these are the same choices faced by firms and their lenders. And they decide which loan products are right for them taking into account how their repayments will be affected by different macroeconomic shocks. These choices are just one of the many ways in which aggregate risks are tradeable, even when individual-specific risks are not.

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