Shape Instabilities driven by defects with different topological charge in Nematic Polymer Networks

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Abstract:
Nematic polymer networks (NPNs) are anisotropic rubbers formed by cross-linked polymeric chains with rod-like mesogenic segments, that, in the nematic phase, align along a non-polar director $\textbf{\emph{n}}$. In NPNs the coupling between nematic order and polymer network is strong enough to constrain the director field to follow elastic deformations. Consequently, variations in the degree of orientation of the mesogenic segments, s, can drive the system out of equilibrium, inducing shape changes.
A key characteristic of nematic systems is the existence of singularities in the director field, known as topological defects, which can be classified by their topological charge m. They play a major role in the self-assembly of biological materials, such as plasma membrane, wood, silk, and the insect cuticles. Developing organisms achieve biological organisation by growing persistent protrusions or depleting material to relieve mechanical stresses originating from the presence of topologically required defects.
This talk aims to provide a mathematical framework describing the out-of-plane shape deformations of initially flat NPN sheets subsumed with a central topological defect and resulting director profile spanning the entirety of the film.
We define an energy associated with these deformations,
consisting of two contributions: an elastic energy term accounting for spatial variations in $\textbf{\emph{n}}$, and a strain-energy function describing the elastic response of the polymer network.
The interplay between nematic elasticity, which seeks to minimize distortions in the director field, and mechanical stiffness, which resists deformation, determines the
resulting morphology.
We analyse the transition to instability of the flat configuration, which represents the systemâ€™s ground state in the absence of nematic ordering. We also characterize the nature of the first buckling modes.