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Abstract:
This work examines the dynamics of avascular tumour growth, focusing on the inelastic distortions that arise from growth and are introduced through a multiplicative decomposition of the deformation gradient tensor. Building on [1], we derive a governing law for the evolution of growth-induced inelastic distortions by linking generalised forces with kinematic descriptors associated with the growth tensor. This formulation, based on the dissipation inequality, reveals the interactions between inelastic distortions and the source and sink terms in the mass balance equations, highlighting the interplay between mechanical stresses and chemical interactions.

A key aspect of this study is the role of the chemicals' non-local behaviour (in time and/or space), which we identify as a significant factor in the growth process [2]. To represent the non-local properties of these chemicals, assumed to be generated by the tumour's complex and multi-scale microenvironment, we introduce integro-differential operators with power-law-type kernels. We then examine how the anomalous behaviour of these chemicals affects key variables governing the system's evolution and offer new perspectives on the challenges of modelling growth processes in heterogeneous biological systems.

References:
[1] Grillo, A. and Di Stefano, S. (2023). Mathematics and Mechanics of Complex Systems, 11(1), 57-86. DOI:10.2140/memocs.2023.11.57.
[2] Almudarra MM, Ramirez-Torres A (2025) Examining avascular tumour growth dynamics: A variable-order non-local modelling perspective. Mathematics and Mechanics of Solids, 30(2):501-526. doi:10.1177/10812865241230269